SOME PECULARITIES OF CONTROL BY FREE-FLYING SPACE ROBOTIC MODULE WITH MANIPULATORS

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ABSTRACT

The paper is devoted to designing of free-flying space robotic module (SRM) with manipulators for International Space Station's (ISS) service. SRM is intended for assembly, external maintenance, inspection and scientific experiments for manned space stations (Ref.1).

Keywords: Space Robotic Module, Optimality, Safety, Predicted Dynamics, Technological Controllability.

1. INTRODUCTION

Among the others requirements to SRM the following two (Ref.2) could be separated: 1) optimality of on board power/ energy consumption; 2) safety of SRM itself and security of ISS and other space objects, to be connected with SRM's activities. From the viewpoint of safety it is very important the predicted dynamics of SRM's motion during its activities. But there exists a problem: different positions of SRM's manipulators change its moment of inertia. Especially it takes place when manipulators grasp some loads which could have different masses and dimensions. So the mass-inertia SRM's characteristics and as consequence the effectiveness of control devices are not known in advance. As a result the SRM's dynamics will be not only optimal but not predicted: so the SRM dynamic oscillations could be not admissible from the viewpoint of SRM's and ISS's safety. Moreover it will be shown that in some positions SRM with manipulators and loads becomes not controllable that is instead of rotation to required direction its manipulators' links could rotate to opposite direction. So the first problem to be solved in the paper could be formulated as: what does it mean the concept of technological controllability for concrete SRM and how to every time moment to calculate the current technological controllability's degree?

On the base of the answer to the first problem in the paper it is proposed the method of calculating the current permissible domain where SRM possesses by technological controllability. The information about the current SRM's degree and permissible domain of technological controllability makes possible to synthesize the algorithms of SRM's and manipulators' control. But it is arising the second problem to be solved in the paper: let the current SRM's degree and permissible domain of technological controllability are known. How to choose the SRM's and manipulators' control algorithms so that the control system dynamics would be predicted? The second problem is solved on the base of adaptive and co-ordinate-parametric control approaches.

The reason for the second problem is the same: the control devices' effectiveness is not known and moreover it changes during the control process. In the paper there will be represented the next solutions:

1. Let the control devices effectiveness is not known but constant. Is it possible to realize the strict time-optimal trajectory for SRM's angular rotation and manipulators' links motion? In the paper there will be represented adaptive algorithm that allows to answer positively and to realize the strict time optimal trajectory without knowing the value of the effectiveness.

2. Let the control devices effectiveness is changed during the motion. In the paper there will be represented the concrete algorithm to realize the quasi time-optimal SRM's motion and some simulation oscillograms for this case.

2. MATHEMATICAL MODEL OF SPACE ROBOTIC MODULE

In common case the SRM's equations of motion can be written as

$$A(q)\ddot{q} + B\dot{q} + Cq + H(q,\dot{q}) = M(q) \qquad (1)$$

where $q \in R^n$ is the generalized co-ordinate vector. The mathematical model (MM) (1) is nonlinear and it is difficult to say some conclusions about SRM as a control object on the base of this MM. But MM (1) could be simplified to the form

$$A(q)\ddot{q} = M(q) \qquad (2)$$

with $H(q,\dot{q}) \approx 0$ for the assumption that the velocity vector $\dot{q}$ is small and $B \approx 0$ for the assumption that the ISS's trajectory is high enough. The MM (2) is more simple than (1) but it is nonlinear nevertheless.

Let us assume that $M(q)$ could be presented as $M(q) = D(q)M$, where $M^T = (M_1, M_2, ..., M_n)$ and the matrix $D(q) = E_n$ for $q = 0$, E_n – is identity matrix. Then the SRM's MM could be written in the form

$$\ddot{q} = R(q)M \tag{3}$$

where $R(q) = A^{-1}(q)D(q)$. Assume that

$$|M_i| \le M_i^{\max}, \; M_i^{\max} > 0 \tag{4}$$

and denote as $M^{\max}$ the control restriction vector

$$M^{\max T} = (M_1^{\max}, M_2^{\max}, ..., M_n^{\max}). \tag{5}$$

3. DEFINITION OF TECHNOGICAL CONTROLLABILITY FOR SPACE ROBOTIC MODULE

We shall call SRM with MM (3) as autonomously technologically (AT) controllable with respect to co-ordinate q_i $(i = \overline{1,n})$ at the position $q = q^*$, if for $q_i(t) = 0$, $\dot{q}_i(t) = 0$, $\ddot{q}_i(t) = 0$ at $(t < t^*)$ and the action $|M_i(t)| = M_i^{\max}$ at $t \ge t^*$ there arises acceleration

$$|\ddot{q}_i(t)| \ge r_i \ne 0 \tag{6}$$

where: 1) it takes place regardless from presence or not presence of others control actions M_j $(j = \overline{1,n};\; j \ne i)$; 2) the signs of $M_i(t)$ and $\ddot{q}_i(t)$ at $t \ge t^*$ are the same.

If at the inequality (6) $r_i \ge r_i^0$, $r_i^0 = const > 0$ $(i = \overline{1,n})$ then we shall call SRM with MM (3) as AT-controllable at the position $q = q^*$ with the degree of AT-controllability equal r_i^0.

We shall call SRM with MM (3) as AT-controllable at the position $q = q^*$ if it is AT-controllable with respect to any co-ordinate q_i $(i = \overline{1,n})$ simultaneously.

We shall call SRM with MM (3) as AT-controllable at the position $q = q^*$ with the degree of AT-controllability equal r^0 where

$$r^{0T} = (r_1^0, r_2^0, ..., r_n^0) \tag{7}$$

if it is simultaneously AT-controllable with the degree of AT- controllability equal r_i^0 $(i = \overline{1,n})$ with respect to the co-ordinate q_i.

We shall call SRM with MM (3) as AT-controllable at the domain $G(q)$ of the generalized co-ordinate space $\{q\}$ if it is AT-controllable at any point $q^* \in G(q)$.

4. NECESSARY AND SUFFICIENT CONDITIONS FOR AT-CONTROLLABILITY OF SRM

Let us consider the equation (3) where $R(q) = (R_{ij}(q))$ is $(n \times n)$-matrix and introduce the matrix $S(q) = (S_{ij}(q))$ where

$$S_{ii}(q) = |R_{ii}(q)|, \quad S_{ij}(q) = -|R_{ij}(q)| \\ (i = \overline{1,n};\; j = \overline{1,n};\; j \ne i). \tag{8}$$

We shall call vector $x^T=(x_1,x_2,\dots,x_m)$ as positive (x>0) (nonnegative ($x \ge 0$)) or matrix $X=(x_{ij})$ $(i = \overline{1,m};\; j = \overline{1,n})$ as positive (X>0) (nonnegative ($X \ge 0$)) if all elements of vector x or matrix X are positive (nonnegative) (Ref.3).

Theorem. Necessary and sufficient conditions for AT-controllability of SRM with MM as (3) for the position $q = q^*$ are the next relations:

1) nonsingularity of the matrix $S(q)$ that is the equality

$$\text{rank}\; S(q^*) = n; \tag{9}$$

2) the existence of at least a single positive solution of the inequality

$$S(q^*)g > 0 \tag{10}$$

where $g \in R^n$.

Let us note that according to the theorem the fact of SRM's AT-controllability at the position $q = q^*$ is determined by only SRM's construction parameters and not by the vector of control restrictions $M^{\max}$ (5) that is SRM at the position $q = q^*$ either AT-controllable or not controllable and at the last case it is not possible to achieve AT-controllability by changing of the vector $M^{\max}$ (5). But if SRM is AT-controllable at the position $q = q^*$ then it is possible to achieve the prescribed degree of AT-controllability r^0 (7) by property choice of the vector $M^{\max} = M_0^{\max}(q^*, r^0)$.

5. SEARCHING OF MINIMAL NECESSARY CONTROL AT THE POINT

Let at the position $q = q^*$ SRM is AT-controllable and it is necessary to search the vector $M_0^{\max}(q^*, r^0)$ that guarantees the degree of AT-controllability at the point $q = q^*$ not less than r^0 with the minimal norm of the vector $M_0^{\max}(q^*, r^0)$. This vector is defined by the equality

$$M_0^{\max}(q^*, r^0) = S^{-1}(q^*)\, r^0. \tag{11}$$

6. SEARCHING OF THE SRM'S AT - CONTROLLABILITY DOMAIN

The boundary surface for the $G(q)$ domain is defined by points where the theorem's conditions are broken that is

$$\det[S(q^*)] = 0,$$
$$g_i R_{ii}(q^*) - \sum_{\substack{j=1 \\ (j \neq i)}}^{n} g_j R_{ij}(q^*) = 0 \tag{12}$$

for any $g_i > 0 \quad (i = \overline{1,n})$.

7. SOLUTION OF SOME TASKS ABOUT SRM'S AT - CONTROLLABILITY

In the Fig. 1 some construction is shown that cold be consider as an example of SRM's plane motion (Ref.1). The vector of generalized co-ordinates includes 5 components: $q_1 = X_0$, $q_2 = Y_0$, $q_3 = J$, $q_4 = a_1$, $q_5 = a_2$. The concrete data are:

$$m_0 = 100 \ kg; \quad I_0 = 40 \ kgm^2;$$
$$m_1 = 3 \ kg; \quad m_2 = 2 \ kg; \quad x_a = 0{,}5 \ m;$$
$$y_a = 0 \ m; \quad r_1 = 0{,}7 \ m; \quad r_2 = 0{,}8 \ m.$$

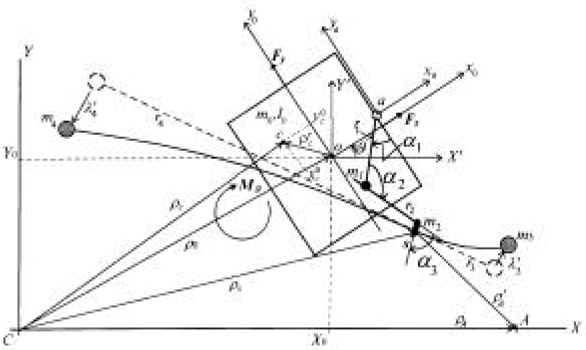

Fig. 1.

For the case of $q_1 = q_1^*$, $q_1^* = const$; $q_2 = q_2^*$, $q_2^* = const$; $q_3^* = q_4^* = q_5^* = 0$; and $r_i^0 = 0{,}1 \ (i = \overline{1{,}5})$ accordingly to (11) the vector $M_0^{\max}(q^*, r^0)$ of minimal norm is

$$[M_0^{\max}(q^*, r^0)]^T = (14{,}07 \quad 11{,}12 \quad 7{,}20 \quad 1{,}14 \quad 0{,}57).$$

In the Fig. 2 the domain of SRM's AT- controllability is shown with the help of cross-sections for $q_5^* = 0$; $q_5^* = 1{,}5$; $q_5^* = 2{,}185$ at Descartes co-ordinate system $0q_3q_4$.

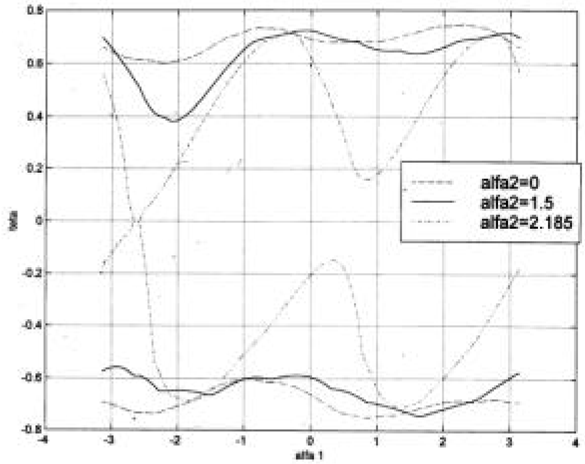

Fig 2.

8. PRESCRIBED QUASI – 0PTIMAL CONTROL BY FREE – FLYING SPACE ROBOTIC MODULE WITH MANIPULATORS

For simplicity we assume that $M_i(t) = k_i \ u_i(t)$, where $u_i(t)$ is the co-ordinate that could be organized with the help of control law and $k_i = const > 0$. Then from equation (3) we receive the relation

$$\ddot{q}_i = k_i R_{ii}(q) u_i(t) + F(q,t), \tag{13}$$

where $|F(q,t)| \le F_0(q)$ and for $q = q^*$ $F_0(q^*) = const$. We denote $k_i R_{ii}(q) = r_i(q)$ and in this paper we assume the case of $F(q,t) \equiv 0$ (the results could be easy extended for the case of $F(q,t) \neq 0$). Then from equation (13) we receive the equation

$$\ddot{q}_i = r_i(q) u_i(t) \tag{14}$$

where the co-ordinate $r_i(q)$ could be named as control effectiveness. The control effectiveness $r_i(q)$ could be changed not only from point to point in the space $\{q\}$ but with the time too that is $r_i = r_i(q,t)$ (Ref.4). So we assume that the value of $r_i(q,t)$ is not known but for the domain $G(q)$ it takes place the inequality

$$r_i^{\min} \le r_i(q,t) \le r_i^{\max}, \tag{15}$$

where $r_i^{\min}$, $r_i^{\max}$ are known constant positive numbers.

Let the task is to change the position q_i from the point $q_i = 0$ to the point $q_i = q_{i\,pr}$, $q_{i\,pr}$= const. If the effectiveness $r_i(q,t)$ is constant for example $r_i(q,t) = r_i^0$, $r_i^0 =$ const > 0 and the value of r_i^0 is known then the solution of the task could be received on the base of the Pontryagin maximum principle (Ref.5) and this solution is well known as the control law

$$u_i(t) = \operatorname{sgn}\left[x(t) - \frac{(\dot{q}_i(t))^2}{2l_i^0}\operatorname{sgn}(\dot{q}_i(t))\right] \tag{16}$$

where $x(t) = q_{i\,pr} - q_i(t)$ and $l_i^0 = \dfrac{r_i^0 M_i^{\max}}{k_i}$.

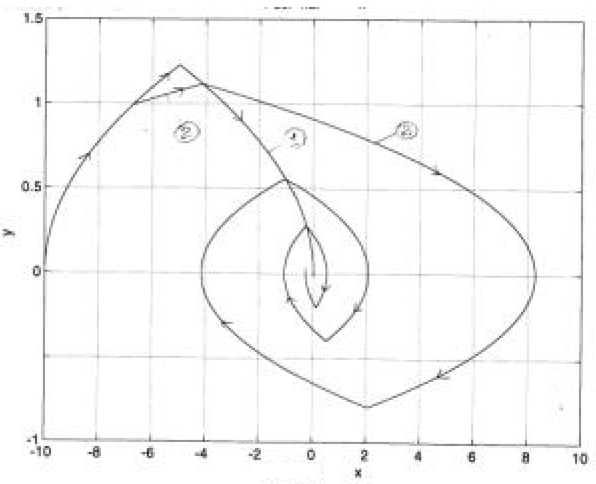

Fig 3.a.

In the Fig. 3.a we can see the time-optimal trajectory 1 on the phase-plane with co-ordinates x and $y = \dfrac{dx}{dt}$ for the case $r_i^0 = 0{,}15$. But on the same figure we see the trajectory 2 when the effectiveness was changed from the value $r_i^0 = 0{,}15$ to the value $r_i = 0{,}08$ at some moment.

The trajectory 2 is not only time-optimal but may be inappropriate from the technical point of view. In paper (Ref. 4) for this case the control law in the form

$$u_i(t) = \operatorname{sgn}\left[x(t) - \frac{(\dot{q}_i(t))^2}{2l_i^{\min}}\operatorname{sgn}(\dot{q}_i(t))\right] \tag{17}$$

is proposed where $l_i^{\min} = \dfrac{r_i^{\min} M_i^{\max}}{k_i}$. This control law guarantees the quasi-optimal trajectory that is shown in Fig.3.b. This trajectory was received for $r_i = 0{,}15$ on acceleration part and $r_i = 0{,}06$ on the deceleration part of trajectory. Comparing trajectory 2 on the Fig. 3.a and trajectory on the Fig. 3.b we can see that quasi-optimal control law guarantees the prescribed dynamics in the sense of absence any motion around the point $(0,0)$ that is the trajectory is completely belonged to one quadrant.

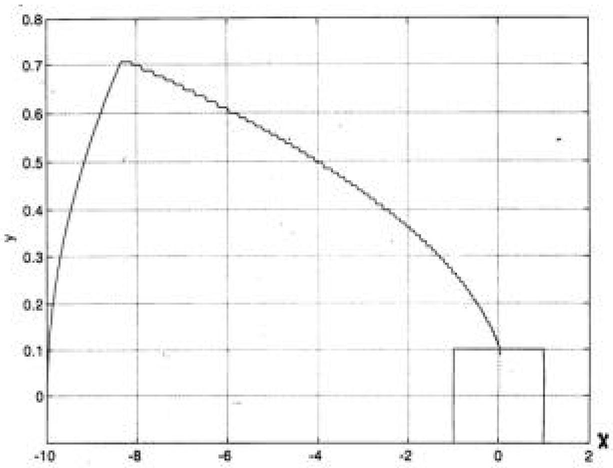

Fig.3.b.

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