Semi-Analytical Guidance Algorithm for Fast Retargeting Maneuvers Computation during Planetary Descent and Landing

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Several improvements in last years, but not enough
Landing uncertainty still imposes strict requirements onto Landing Site choice

Could allow the reaching of more scientifically relevant places
It involves the presence of an Hazard Detection and Avoidance System (HDA)

Autonomous, Precise and Safe Landing Capability
Is a key feature for the next space systems generation.
Development of a GUIDANCE ALGORITHM

- Capable to dynamically recompute and correct the landing trajectory during the descent

Requirements

- Computational Efficiency
- Maximum Attainable Landing Area (maximize the probability to find a safe landing site)
- Landing Accuracy

Reference Scenario (for simulations)

- ESA Lunar Lander
  - Demonstration of Safe Precision Landing Technology as part of preparations for participation to future human exploration of the Moon
Parking Orbit (LLO 100km)

Deorbiting (DOI) & Coasting (100x15km)

Main Brake (PDI)
- **Constant Thrust**
- **High Gate** (HG): Nominal Landing Site comes into Sensors FoV

Approach
- **Approach Gate** (AG): Hazard Detection System comes into operation
- Once updated the Hazard Map, **Thrust is reduced**, and **Retargeting could be commanded**
- **Throttleable Thrust** after first Retargeting
- **Low Gate** (LG): Last chance of Retargeting

Terminal Descent (TD)
- Starts at 30m altitude (**Terminal Gate**, TG)
- Vertical descent at **Constant Speed** -1.5m/s until Touch Down

Adaptive Guidance operates from Retargeting to Terminal Descent
**Reference Trajectory**

- **Planar** Reference Trajectory (from a preliminary optimization)
- From Powered Descent Initiation (PDI) to Terminal Gate (TG)
- Starting point for Retargeting

**Mass at Touchdown:** 816 kg (in line with ESA Lunar Lander).
Altitude at thrust reduction (first chance of retargeting): 2000 m

Hazard Detection System operation field:
- Altitude < 3000 m
- NLS View angle < 45 deg

Hazard Detection available time before thrust reduction: 21.4 s
Adaptive Guidance: Dynamics

- Constant Gravity Field with flat ground (small altitude compared to planet’s radius)
- No aerodynamic forces considered (also with low density atmosphere, speed < 100m/s)

\[ \dot{\mathbf{r}}(t) = \frac{T(t)}{m(t)} + \mathbf{g} \]

\[ \dot{m}(t) = -\frac{T(t)}{I_{sp} g_0} \]

- Thrust vector expressed in Euler Angles and Thrust Magnitude (Control Variables)

\[ \mathbf{T}(t) = -T(t) \begin{bmatrix} \cos \psi(t) \cos \theta(t) \\ \cos \psi(t) \cos \theta(t) \\ -\sin \psi(t) \end{bmatrix} \]
Thrust dependent acceleration:
\[ a(t_0) = \begin{bmatrix} -T_0 \frac{\cos \psi \sin \theta}{m} - g_M \\ -T_0 \frac{\cos \psi \cos \theta}{m} \\ T_0 \frac{\sin \psi}{m} \end{bmatrix} \]

- Known Position and Speed:
  - \( r(t_0) = r_0 \)
  - \( v(t_0) = v_0 \)
- Thrust dependent acceleration:

Desired Position and Speed:
- \( r(t_{ToF}) = \begin{bmatrix} 30 \\ 0 \\ 0 \end{bmatrix} \) m
- \( v(t_{ToF}) = \begin{bmatrix} -1.5 \\ 0 \\ 0 \end{bmatrix} \) m/s

Vertical Attitude:
- \( a(t_{ToF}) = \begin{bmatrix} \text{FREE} \\ 0 \\ 0 \end{bmatrix} \)

**Final Constraints**

**Initial Constraints**

2 Variables: \( T_0, t_{ToF} \)

17 Boundary Constraints
**Adaptive Guidance: Descent Profile**

- **Polynomial Acceleration Profile**
- **Minimum degree needed** to satisfy Boundary Constraints
- Depends on 2 variables: \( x = \begin{bmatrix} T_0 \\ t_{ToF} \end{bmatrix} \)

**Acceleration Profile**

\[
\begin{align*}
    a_x(t) &= a_{0x} + C_{1x} t + C_{2x} t^2 \\
    a_{y/z}(t) &= a_{0y/z} + C_{1y/z} t + C_{2y/z} t^2 + C_{3y/z} t^3
\end{align*}
\]

**Thrust-to-Mass Ratio**

\[
P(t) = \frac{T(t)}{m(t)} = \begin{bmatrix} a_x + g_M \\ a_y \\ a_z \end{bmatrix}
\]

**Mass vs Time Trend**

\[
m(t) = m_0 \exp \left( - \int_{t_0}^{t} \frac{\|P(t)\|}{I_{sp}g_0} dt \right)
\]

Solved with **Pseudospectral Methods**

**Guidance Profile**

\[
\begin{align*}
    \theta(t) &= \tan^{-1} \left( \frac{1}{\|T\|} \frac{T_x}{T_y} \right) \\
    \psi(t) &= \tan^{-1} \left( \frac{1}{\|T\|} \frac{T_z}{\sqrt{T_x^2 + T_y^2}} \right) \\
    \phi(t) &= 0 \\
    T(t) &= \|T(t)\|
\end{align*}
\]
Limited Search Domain

- Initial Thrust Magnitude
- Time-of-flight

Additional Constraint

- Final Mass

Path Constraints

- Available Thrust
- Available Control Torques
- Glide-Slope Constraint

Problem Formulation

\[ \min f(x) = m_0 - m(t_f) \] such that
\[ \begin{aligned}
    x_L &\leq x &\leq x_U \\
c_L &\leq c(x) &\leq c_U \\
\end{aligned} \]

Path Constraints are evaluated \textbf{discretely} with \textbf{Pseudospectral Techniques}
GuidALG: Compass Search Method

- **Direct Optimization method**: Cost function treated as “Black Box”
- **Low Computational Cost**

**Step 1 – FEASIBILITY**

Unconstrained Compass Search on the **Feasibility Function**:

\[ F(\bar{x}) = \sum_{i=0}^{Nc} \max(0, \bar{c}_i(\bar{x})) \]

Generalized Constraints vector:

\[ \bar{c}(\bar{x}) = \begin{bmatrix} c_L - c(\bar{x}) \\ c(\bar{x}) - c_U \\ 0 - \bar{x} \\ \bar{x} - 1 \end{bmatrix} \]

**Step 2 – OPTIMALITY**

- **Unconstrained Compass Search on Modified Cost Function**:

\[ \phi(\bar{x}) = f(\bar{x}) + \eta \text{sgn}(F(\bar{x})) \]
The algorithm precision is affected by the choice of pseudospectral order of approximation $N$ in mass calculation:

- As $N$ increases:
  - Computation Time Increases
  - Precision Increases

Performances estimated with Monte Carlo simulations:

- Random diversion $\pm 2000$ m from Nominal Landing Site
- 100000 samples for each value of $N$
- From $N \geq 20$ Error is negligible.
Attainable Area from Nominal Path at different altitude

- 4000x4000 m Test Area Centered on Nominal Landing Site
- 100000 samples Monte Carlo Analysis

Adaptive Guidance: **Divert Capability**

Downrange Direction

- Feasible
- Feasible, Suboptimal
- Infeasible

4000 m

2000 m

1500 m

1000 m

500 m

100 m

200 m

200 m

ASTRA 2013 - ESA/ESTEC, Noordwijck, the Netherlands
• Solution compared with alternative methods:
  • GuidALG, (proposed method)
  • Polynomial formulation with nonlinear optimization solver (SNOPT)
  • Direct Collocation Method (more accurate, more slow)

• Found solution is an approximation of the optimum.
**Actuators Model**
- **Ideal Actuators**
- ACS Thruster: constant control torques, ±40 Nm on each axis
- **Throttleable Main Thrust** 1000÷2300N

**Lander Dynamics**
- 6DoF Model
- Thrust misalignment Disturbance Torques

**GUIDANCE**

**CONTROL law**

Control Update Rate: **20 Hz**

**Navigation (Error Model)**

**Commanded DIVERSION**

**Closed-Loop Landing Simulation**

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Navigation System emulated by Errors added to States

**Attitude**
- **Inertial Measurement Unit** (IMU), calibrated with Star Trackers just before PDI

**Position and Speed**
- **Visual-Based Navigation System**
- Make use of Laser/radar Altimeter to rescale Images
- Zero-mean Gaussian Error
- Standard Deviation Linearly Dependent by Altitude

**IMU Performance Properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>UoM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Factor</td>
<td>1</td>
<td>Ppm</td>
</tr>
<tr>
<td>Misalignment Error</td>
<td>170</td>
<td>μrad</td>
</tr>
<tr>
<td>Bias Error</td>
<td>0.005</td>
<td>deg/h</td>
</tr>
<tr>
<td>ARW Noise Density</td>
<td>0.005</td>
<td>deg/√h</td>
</tr>
</tbody>
</table>

**Altitude**

<table>
<thead>
<tr>
<th>Altitude</th>
<th>2000 m</th>
<th>0 m</th>
<th>Confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position Error</td>
<td>±25 m</td>
<td>±0.1 m</td>
<td>1σ</td>
</tr>
<tr>
<td>Speed Error</td>
<td>±0.4 m/s</td>
<td>±0.1 m/s</td>
<td>1σ</td>
</tr>
</tbody>
</table>

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Landing Simulation: Guidance & Control

Trajectory Update
- Whenever a Retargeting occurs
- Every 5 seconds (minimize dispersions)

Guidance/Control Interface
- Target Quaternions
- Target Angular Speed
- Target Thrust Magnitude

PID

Theoretical Control Torques

PWPF Modulation

ACS Thrusters Activation Scheme

Hazard Detection
- TLS Coordinates

Navigation
- States Estimation

States Estimation

Guidance Profile

$T(t)$ to Main Engine

Navigation

States Estimation

Guidance Profile

Theoretical Control Torques

PWPF Modulation

ACS Thrusters Activation Scheme

Hazard Detection
- TLS Coordinates
Landing Dispersion at different Target Landing Site
100 Samples series of Monte Carlo Analysis

Landing precision appears to be independent by the magnitude of the requested diversion.
Landing Dispersion at different Position Initial Error Standard Deviation (STD)
100 Samples series of Monte Carlo Analysis

- Dispersion one order of magnitude lower with exact states estimation;
- Landing precision is mainly affected by Navigation errors.
Navigation Camera

- Relative navigation by Landmaks tracking
- Tracked Landmarks pass continuously into FoV, allowing relative navigation

Hazard Detection Camera

- Target LS must be into FoV for Hazard Map update
- Camera Aperture Angle: 50°–70° deg
- Visibility on TLS lost if Sightline-TLS angle >25°–35° deg
- Oscillatory movement causes initial loss of visibility
- Visibility recovered in 2nd half trajectory
Conclusions

Low Computational Cost Obtained

- Computation Time <100ms in Matlab® language on a PC
- Compatible with Control Update Rate during a real Landing

Wide attainable area even with a simply optimization method

Loss of Divert Capability when nominal trajectory requires High Thrust

- Maximum Attainable Area when Retargeting is ordered in low thrust phases

Navigation Errors play a major role in determining the accuracy at Touchdown
Future Developments

Improve Retargeting Strategy

- Dual Retargeting

Increase Polynomial Order

- More Opt. Parameters
- Add Constraints

Tradeoff on Opt. Algorithm

Integration with Visual-Based Navigation and Hazard Detection Systems

Hardware-in-the-loop Testing
Thank you for your attention
Pseudospectral Discretization

- Function discretized with its values at **Chebyshev-Gauss-Lobatto (CGL) points** \( \tau_k \):
  \[
  \tau_k = -\cos \left( \frac{\pi k}{N} \right), \quad k = 0, \ldots, N
  \]

- Function shifted on computational domain:
  \[
  p(t), \quad t \in [t_{ini}, t_{end}]
  \]
  \[
  p(\tau) = p(t(\tau)), \quad t(\tau) = \frac{(t_{end} - t_{ini})\tau + (t_{end} + t_{ini})}{2}
  \]
  \[
  p = [p_0, p_1, \ldots, p_k]^T, \quad p_k = p(t(\tau_k)), \quad k = 0, \ldots, N
  \]

- The function can be evaluated at each time instant with a polynomial approximation of the form:
  \[
  p(t) \approx \hat{p}(t) = \sum_{k=0}^{N} p_k \varphi_k(t(t))
  \]
  \[
  \varphi_k(t(t)), \quad k = 0, \ldots, N \text{ Lagrange interpolating polynomials of order } N.\]

B1

ASTRA 2013 - ESA/ESTEC, Noordwijk, the Netherlands
• **Cebyshev Differentiation Matrix**

\[
\frac{dp_k}{d\tau} = \frac{d}{d\tau} \left( \sum_{j=0}^{N} p_j \phi_j(\tau_k) \right) = \sum_{j=0}^{N} (D_N)_{kj} p_j
\]

\[
(D_N)_{kj} = \begin{cases} 
  c_i \frac{-1^i+j}{c_j \tau_i - \tau_j}, & i \neq j, \\
  \frac{-\tau_j}{2(1 - \tau_j^2)}, & 1 \leq i = j \leq N - 1, \\
  \frac{6}{2N^2 + 1}, & i = j = 0, \\
  \frac{-6}{2N^2 + 1}, & i = j = N.
\end{cases}
\]

CGL points from -1 to 1. Differentiation Matrix Correction.

\[
\tilde{D}_N = -D_N
\]

\[
c_j = \begin{cases} 
  1, & j = 0, N \\
  2, & 1 \leq j \leq N - 1
\end{cases}
\]

• From Computational domain to real domain:

\[
dt = \frac{t_{end} - t_{ini}}{2} \\
d\tau \Rightarrow \frac{dp}{dt} = \frac{2}{t_{end} - t_{ini}} \frac{dp}{d\tau}
\]
Clenshaw-Curtis Quadrature

- Clenshaw-Curtis Quadrature
- The integral is discretized to a finite sum:

\[
\int_{-1}^{1} p(\tau) d\tau = \sum_{k=0}^{N} w_{ck} p_k
\]

\[
dt = \frac{t_{end} - t_{ini}}{2} d\tau \Rightarrow \int_{t_{ini}}^{t_{end}} p(t) dt = \frac{t_{end} - t_{ini}}{2} \sum_{k=0}^{N} w_{ck} p_k
\]

- \( w_{ck} \) Weights of Clenshaw Curtis Quadrature.