A Unified Approach to Scheduling and Resource Analysis for the Galileo Mission

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Abstract. Scheduling and Resource Analysis have been typically seen as different tasks under different disciplines as Operations Research and Artificial Intelligence. Only very recently some efforts have been done to try to unify them in the broadest sense. However, for some problems (as the one introduced herein) it is feasible to tackle with both problems in a consistent and efficient manner even if it is related to the complex case of time resource-constrained multiple projects management problem.

1 Introduction

GALILEO will be an independent, global European-controlled satellite-based navigation system. The GALILEO system will consist of a MEO constellation of 27 operational satellites (plus 3 in-orbit spare satellites) monitored and controlled by a Ground Segment providing also the capability to detect satellite or system malfunctions and broadcast real-time warnings (integrity messages). Each operational satellite will broadcast a set of navigation signals carrying clock synchronisation, ephemeris, integrity and other data, depending on the particular signal. A user equipped with a suitable receiver will be able to determine his position to within a few metres when receiving signals from visible GALILEO satellites. GALILEO will thus provide an infrastructure for a wide range of guaranteed services to users equipped with receivers meeting GALILEO specifications.

1.1 GALILEO as a major technological, economic and political challenge

Like other major European projects such as the Airbus or Ariane, GALILEO is a technological advance likely to revolutionise society in the same way that the mobile phone has done in recent years while also promoting the development of a new generation of universal services.

GALILEO will afford considerable advantages in many sectors of the economy. In road and rail transport, for example, it will make it possible to predict and manage journey times, or, thanks to automated vehicle guidance systems, help reduce traffic jams and cut the number of road accidents. However, although transport by road, rail, air and sea is the example most frequently quoted, satellite radionavigation is also increasingly of benefit to fisheries and agriculture, oil prospecting, defence and civil protection activities, building and public works, etc. In the field of telecommunications, allied with other new technologies such as GSM or UMTS, GALILEO will increase the potential to provide positioning information as well as to provide combined services of a very high level.

Having control of the satellite constellation technology which is central to the system means having control of the many industrial applications possible thanks to satellite positioning. Europe cannot afford not to become involved in what, it is already clear, will be one of the main sectors of industry in the twenty-first century. That would mean becoming dependent on systems and technologies developed outside Europe for applications vital to the running of the society of tomorrow.

1.2 GALILEO Phase C0 Scope and Objectives

Phase C0 addresses the preliminary design up to the Segments Preliminary Design Review (PDR) for the development and validation of the Galileo Space and Ground Segments. During this phase the segment requirements will be consolidated, the preliminary segment architectures will be defined by decomposition of the segments into its main architecture elements, and by defining internal and external interfaces and element requirements.

1.3 GALILEO Navigation and Integrity Functions

GALILEO as a second generation Global Navigation Satellite System (GNSS) offers a high level of reliability in the broadcast of navigation information, allowing the safe use of the navigation signals for critical (e.g Safety of Life) applications. For that purpose on one hand the navigation data content is qualified beforehand by means of the, so called, Signal-in-Space Accuracy (SISA), and on the other hand the navigation signals are monitored on real time to detect and flag any deviation respect to the predicted behaviour on a satellite per satellite basis (integrity function).

1.3.0.0.1 Navigation Service and Signal-in-Space Accuracy (SISA) The GALILEO System will broadcast for each satellite navigation data set, containing the ephemeris and clock model, an associated quality indicator, suitable as input for the positioning error statistical characterisation (e.g. Protection Levels computation). Such quality indicator has been named as the Signal-in-Space Accuracy (SISA) parameter per satellite.

SISA is a qualifier to characterise of the Orbit and Clock Determination and Prediction process.

1.3.0.0.2 Integrity Service and Signal-in-Space Monitoring Accuracy (SISMA) Integrity in its general sense is the ability of a navigation system to provide timely and valid warnings to the users when their position fails to meet certain margins of accuracy.

In case of a system failure a bias will be introduced in the range measurements. This bias needs to be detected by the system. For that reason, a ground integrity system will be deployed for checking whether the ranging measurements are biased. If the bias is above a certain Integrity Threshold an alert will be raised to the user. This alert is provided in the form of an Integrity Flag (IF) for a specific satellite. The concerned satellite is to be discarded in the Protection Level computation and user positioning solution.

The GALILEO system through the IF will protect the user against satellite orbit and clock errors, signal generation errors and errors in the monitoring system (insufficient measurements, message uplink problems).

The IF is based on the short-term observation of the variations in satellite clock and ephemeris, measured by the GALILEO Sensor Stations, and computed at the Integrity Processing Facility. It will be possible to provide integrity alerts every second.

The qualifier of the Integrity Flag is the so-called Signal-in-Space Monitoring Accuracy (SISMA) parameter. This parameter will be made available to the users. SISMA provides the resolution of the integrity monitoring system, to detect the satellite residual error in the range domain for the worst user location.

1.4 ULS Scheduling and ULS/GSS Network Analysis

Henceforth, one of the major challenges during phase C0 is a comprehensive understanding of the Galileo mission. Simulators, prototypes and other software will be developed during this phase to aid engineers and scientists at ESA to consolidate requirements. One of this tools is the Galileo Analysis and Scheduling Tool (GAST) to be produced by Deimos Space and shall be used by ESA in order to:

- To schedule and analyse the availability of Up-Link Stations (ULS) and antennas over a configurable period of time:
  - To schedule a contacts plan of the up-link operations considering a given list of ULS sites and antennas per site and the constellation subset for integrity and taking into account all the different types of information to uplink and the different constraints described below.

    Instead of simulating the data volumes to uplink to the Galileo spacecrafts, another approach is used: to specify the contact times. The longer they are, the largest the volume they simulate.
  - To analyse the ULS location and number of antennas per site, as well as to re- assess the impact of modifying the locations, number of antennas, or antenna parameters such as the masking angle.
- It shall also:
  - Derive the constellation subset for integrity uplink when the ULS sites are defined such that the integrity constraint is satisfied: a minimum of 2 independent integrity paths have to be provided to any user anywhere and anytime. In order to be fully resistant to any single failure in the system (either satellite, ground antenna, communications link or a complete site), at least 3 independent integrity paths between the Integrity Processing Facility (IPF) and the user will have to be provided to any user anywhere and anytime.
  - Study the Galileo Sensor Station (GSS) sites location such that SISMA is optimised as well as to re-assess the impact of modifying the GSS locations or number. Simultaneously, the application has to be flexible enough to allow the operator to specify different algorithms for the computation of SISMA.

However, these issues are out of the scope of this paper.

Therefore, restricting our attention to the ULSs, two different components are easily identified:

- *Scheduling*: once the resources are well defined, the GAST shall come out with a feasible contact plan with the Galileo constellation which optimises the usage of the ULS sites and its antennas.
- *Resource analysis*: in case the scenario considered does not provide the minimum amount of resources, the GAST shall come out with a report of the minimum number of resources (ULSs and/or antennas) to include in order to derive a feasible optimised schedule.

Therefore, GAST is a scheduling and resource analysis tool that shall aid at designing the Galileo constellation and the antennas network, by running it as many times as necessary during phase C0 under different scenarios.

Both problems, scheduling and resource analysis, are intimitely related to different disciplines as Operations Research (Taha 1997) and, in a more general sense, to Artificial Intelligence. From the perspective of the latter, these problems have been formulated as Constraint Satisfaction Problems (CSPs) where different techniques have been tested;

also some evolutionary computation techniques have been tried as neural networks (Cavalieri and Mirabella 1996) or genetic algorithms (Özdamar 1999; Jensen 2003) and even knowledge management has been used for solving this type of problems (Isaai and Cassaigne 2001). On the other hand, these problems have been treated as being different and only very recently some researchers are trying to relate each other in specific domains (Zaffalon *et al.* 1998) or from a general point of view (Bar-Noy *et al.* 1999). Regarding the domain, this work can be related to the scheduling of the Deep Space Network at the Jet Propulsion Laboratories as described in (Gratch and Chien 1996) but instead of devising a system that can use different heuristics, the following idea is introduced: combining heuristic single-agent search algorithms with constraint satisfaction rules for solving both cases simultaneously.

The paper is arranged as follows: the next section defines the problem describing its resources and constraints. Next, in the same section, a solution is proposed by describing the goal definition, cost function, heuristic function, constraint propagation rules and feasibility rules and, finally, some relevant conclusions in section 3.

2 GALILEO Analysis and Scheduling Tool

This section discusses the inner details of the GAST.

2.1 Resources definition

The scheduling and resource analysis process shall take into account the following resources that define uniquely a scenario:

- *Satellites*. The Galileo constellation is composed of 27 spacecrafts whose configuration is defined as a Walker constellation: total number of satellites; number of planes, with evenly-spaced nodes; phasing parameter, describing the relative angular spacing between spacecrafts in adjacent planes and the orbit inclination in degrees. Although the Walker constellation considers orbits as circular trajectories so that the eccentricity can be assumed to be 0, the orbit propagator implemented could also accept arbitrary values for this keplerian element.

  The number of satellites is denoted with N_s.

  Satellites are characterized with the so-called project requirements that define a set of constraints over them. In a typical scenario the following constraints are defined: the duration of the contact events, t^C, and the minimum and maximum allowed time elapsed between the start times of two successive contacts with the same spacecraft, t^S_{min} and t^S_{max} respectively.

  On the other hand, the GAST allows the operator to simulate potential unavailabilities (e.g., satellite malfunctioning) and satellites that are irremediably lost. The former is simulated specifying the unavailability period within the current makespan and the latter specifying only the start time of the unavailability.

- *Up-Link Stations*. Its role within the Galileo Ground Segment is to uplink the Galileo mission data. Each site consists mainly of: a main equipment room; a number of C-band uplink antennas and a VSAT (Very Small Aperture Terminal) depending upon the communication capabilities.

  Let us denote the number of up-link stations with N_u.

  Every up-link station is expected to have the following geographical attributes: latitude, longitude and height. On the other hand, every up-link station is constrained to have a number of antennas co-located between 0 and 10 —it is worth noting that the upper bound currently considered by ESA is four. On the other hand, different types of up-link stations to be employed are being currently considered including ESA and non-ESA sites. Therefore, ULSs are characterized with a priority value ranging from 0 (the lowest value) to 10 (the highest value. $p(U_i)$ denotes the priority of the site U_i) by the operator.

  Regarding the resource analysis functionality of GAST, the system shall select the minimum number of ULSs from a specified list of potential sites (characterized only with the latitude, longitude and height) such that a contact plan becomes feasible. The number of antennas per site is undefined and they are only defined with a masking angle that equals 5 degrees by default.

- *Antennas*. They up-link the signal generated by Galileo Ground Segment that embraces: navigation data; search and rescue return channel data; navigation related service data; public regulated service data and Galileo originating integrity data and, maybe, up to 5 regional integrity data streams.

  The system consists of N_a antennas.

  Obviously, antennas are characterized with a masking angle (that equals 5 degrees by default) and an elevation value (with a precision of 0.5 degrees) for every azimuth. These parameters (along with the latitude, longitude and height of the ULS they belong to) shall be used for computing the AOS/LOS (Acquisition of Signal/Lost of Signal) of every antenna with every satellite.

  Antennas that do not currently exist are qualified with priority 0. It shall be guaranteed that GAST will consider these antennas only in case there is no other way to derive a feasible contact plan.

  Finally, antennas are subjected to maintenance (and other) operations that disqualify them for up-linking signals. These operations, when being planned to happen within the makespan shall be specified with the start and stop time of the unavailability.

2.2 Goal definition

The GAST shall generate contact plans that aim at minimizing the number of antennas involved at the time the global priority of the ULSs employed is maximized and all the constraints aforementioned are met.

The global priority, P, is defined as:

$$P = \sum_{i=1}^{N_u} p(U_i) \quad (1)$$

In other words, the global priority is defined as the sum of the priorities of the N_u ULSs considered.

2.3 Cost function

A node (or partial solution) n is said to have a cost $c(n)$ which is defined as a tuple $[c_0, c_1, \ldots, c_{10}]$ where c_i is defined as the number of contacts with antennas belonging to the same or different sites such that their priority equals i, e.g. c_0 is the number of contacts with antennas *to build* in order to derive a feasible contact plan; c_1 is the number of contacts with *existing* antennas of priority 1; c_2 is the number of contacts with *existing* antennas of priority 2, and so on.

Although the goal definition shown above might seem to imply a bi-dimensional cost function (the global priority and the number of antennas) that leads to a multicriteria problem (Mandow and de la Cruz 2001; 2003), a complete ordering criteria can be defined to turn the problem into a single optimization problem which makes use of the child ordering technique:

$$[c_0, c_1, \ldots, c_{10}] \prec [c'_0, c'_1, \ldots, c'_{10}] \quad (2)$$

if and only if:

$$c_i = c'_i \text{ and } c_k \leq c'_k \forall i, 0 \leq i < k \leq 10 \quad (3)$$

This ordering is known as the *lexicographical order*.

2.4 Heuristic function

A heuristic function, $h(n)$, returns the estimated cost of the cheapest path from the state node n to a goal state (Pearl 1984)[1]. In this case, $h(n)$ is expected to return a tuple $[h_0, h_1, \ldots, h_{10}]$ where h_i is the minimum number of antennas of priority i that are necessary to up-link the Galileo messages to all the spacecrafts. Therefore, the expected cost of the best solution below the state node n can be computed as $[c_0, c_1, \ldots, c_{10}] + [h_0, h_1, \ldots, h_{10}] = [c_0 + h_0, c_1 + h_1, \ldots, c_{10} + h_{10}]$.

A heuristic function is said to be *admissible* if it returns a cost which is less or equal than the optimal cost, C^*. In other words, $h(n)$ is said to be *admissible* if and only if:

$$[h_0, h_1, \ldots, h_{10}] \prec [C^*_0, C^*_1, \ldots, C^*_{10}] \quad (4)$$

where C^*_i is the optimal number of antennas of priority i. It has been shown that admissible heuristic functions exhibit very interesting properties like: first, they do never allow the search algorithm to expand a node that has no chance to lead to an optimal solution (Hart *et al.* 1968) and secondly, they force the search algorithm to exit with an optimal solution if any exists (Pearl 1984).

The heuristic function proposed has been obtained by *constraint relaxation* (Pearl 1984; Hansson *et al.* 1992) by assuming that contacts can happen anytime —even out of the scope of the coverage events what is clearly unfeasible. It first computes the availability of the antennas throughtout the makespan. it can be easily shown that an antenna a_i can be used up to:

$$d_i \times \left[1 + \frac{T_1 - t}{t^C}\right] \quad (5)$$

from a given time t up to end of the horizon T_1, where d_i is the duty cycle of the antenna a_i. On the other hand, the minimum number of antennas required for the remaining makespan of every satellite s_i is:

$$\sum_{i=1}^{N_s} \frac{T_1 - t_i}{t^S_{max}} \quad (6)$$

where t_i is the start time of the last contact of the satellite s_i. Therefore, a vector H^+ representing the availability of antennas is initialized to 0 for all priorities between 0 and 10. For every antenna within the scenario (considering even antennas in the list of antennas that might be built for network analysis purposes), its availability is computed taking into account its duty cycle according to equation (5). Next, the minimum demand of all satellites is computed as the sum shown in equation (6) for every satellite. So, the demand of antennas (expressed as a scalar value) can be assigned to the antennas availability (expressed as a vector) by considering first the antennas with higher priority until the demand becomes empty or the availability becomes null. In the second case, a deadlock has been found and the search algorithm is allowed to backtrack safely.

It is worth noting that this heuristic function:

- Tries to include always the antennas with the highest priority. In other words, it tries to avoid employing antennas with priority 0 as much as possible. Therefore, it is guaranteed that in case the algorithm exists with a solution that employs antennas *to be built*, no other solution exists.
- It does take into account the duty cycle of every antenna considered in the current scenario.
- It gives priority to nodes n with the shorter remaining makespan unless another solution exists that employs antennas with highest priority.

[1] There is also another type of *heuristics* which are formulated as domain-dependant strategies that return other states (Georgeff 1983) but they are not considered in this paper.

- It is admissible, since it returns the minimum number of antennas to use for the remaining makespan of every satellite s as computed in equation (6).

2.5 Search algorithm

The two major issues to consider when choosing the search algorithm are:

- It shall be *admissible*. In other words, it shall exit with optimal solutions if any exist when guided by admissible heuristic functions like the one shown in the previous section.
- It shall aim at finding a global solution very fast which could be refined later progressively.

Of all the optimization single-agent search algorithms described in the specialized bibliography the best choice is DFBnB (Depth-First Branch-and-Bound) that is an algorithm belonging to the class of branch-and-bound search algorithms (Balas 1968; Mitten 1970; Kumar 1987). This algorithm is especially well suited for our case since the depth of the search tree is bounded so that a solution can be found rapidly (Zhang and Korf 1995).

On the other hand, this single-agent search algorithm allows the operator to interrupt it. In case the search algorithm has a feasible solution at hand it can propose the operator to exit with it. Otherwise, it can progress from it refining its value towards the global optimum: $[C_0^*, C_1^*, \ldots, C_{10}^*]$.

The following sections describe several constraint propagation rules used to derive a feasible contact plan for any combination of satellites, stations, antennas and different values of t^C, t^S_{min} and t^S_{max}.

2.6 Selection rule

The search algorithm starts by considering the coverage events of every antenna. The Acquisition of Signal (AOS)/Lost of Signal (LOS) of every antenna/satellite is obtained by considering the horizon profile of every antenna and the orbit parameters of every satellite. Let us denote $t^{s,a}_{i,0}$ as the start time of the i-th *coverage* event of the satelite s with the antenna a. Analogously, let us denote the stop time of the i-th coverage event of satellite s and antenna a as $t^{s,a}_{i,1}$. Similarly, $t^{s,a}_j$ denotes the start time of the j-th *contact* event between satellite s and antenna a. Due to the project requirements, $t^{s,a}_j$ is bounded between $t^{s,a'}_{j-1} + t^S_{min}$ and $t^{s,a'}_{j-1} + t^S_{max}$ where a' is the antenna the satellite contacted before. Taking into account that the start time of the $(j-1)$-th contact between satellite s and antenna a' is also constrained to fall within the interval $[t^{s,a'}_{j-1,0}, t^{s,a'}_{j-1,1}]$ (as it will be promptly shown), the j-th contact between satellite s and antenna a is bounded within $[t^{s,a'}_{j-1,0} + t^S_{min}, t^{s,a'}_{j-1,1} + t^S_{max}]$. Therefore, when scheduling the j-th contact, the following **selection rule** shall be applied: for every antenna a, select the coverage events with satellite s that intersect with the aforementioned interval.

In other words, the DFBnB search algorithm branches considering for a given satellite all the coverage events picked up with the selection rule.

2.7 Association rule

The next step consists of creating opportunity windows for the start time of the j-th contact between satellite s and antenna a. An interval $[t_a, t_b]$ is said to be the **opportunity window** of the start time of an association between satellite s and antenna a if and only if it can be proved that $t^{s,a}_j$ necessarily has to start within that interval. It is easy to realize that a new link operation between satellite s and antenna a is feasible if and only if:

$$\begin{aligned} t^{s,a}_{j-1} + t^S_{max} &\geq t^{s,a}_{i,0} \\ t^{s,a}_{i,1} - t^C &\geq t^{s,a}_{j-1} + t^S_{min} \\ t^{s,a}_{i,1} - t^C &\geq t^{s,a}_{i,0} \end{aligned}$$

The first and third constraints guarantee that the opportunity window for the start time of the link will start within $[t^{s,a}_{i,0}, t^{s,a}_{i,1}]$, i.e., within the coverage event. The second condition ensures the link operation will have an opportunity for having a span equal to t^C. Obviously, they can be overwritten simply as:

$$\begin{aligned} t^{s,a}_{j-1} + t^S_{max} &\geq t^{s,a}_{i,0} \\ t^{s,a}_{i,1} - t^C &\geq max\left(t^{s,a}_{j-1} + t^S_{min}, t^{s,a}_{i,0}\right) \end{aligned}$$

Thus, the initial opportunity window for this association is obtained by applying the following **association rule**: when associating satellite s with antenna a, the opportunity window of the start time of the link operation shall belong to the interval $[t^{s,a}_{j,0}, t^{s,a}_{j,1}]$ defined as follows:

$$\begin{aligned} t^{s,a}_j \in &\left[max\left(t^{s,a}_{i,0}, t^{s,a}_{j-1} + t^S_{min}\right),\right. \\ &\left. min\left(t^{s,a}_{i,1} - t^C, t^{s,a}_{j-1} + t^S_{max}\right)\right] \end{aligned}$$

2.8 Forward propagation rule

Once an association between satellite s and antenna a has been actually done, new temporal constraints shall be propagated to ensure that no other satellite s' is simultaneously linked to the same antenna a. In other words, a method shall be devised to guarantee the global feasibility of a set of links $s - a$. This method shall be applied to (virtually) all satellites linked with the same antenna a. An expected side effect of this method is that it shall restrict further the allowed opportunity windows of every link such that the new set of links become feasible. In the incoming discussion, it is always assumed that every time an opportunity window $[t^{s,a}_{j,0}, t^{s,a}_{j,1}]$ is

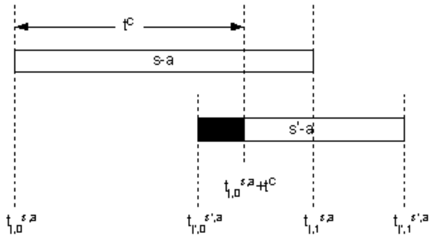

Figure 1: Intersection of the opportunity windows $s - a$ and $s' - a$ (I)

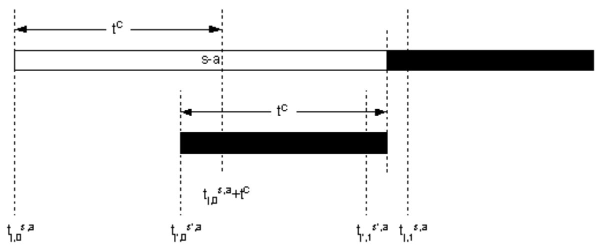

Figure 2: Intersection of the opportunity windows $s - a$ and $s' - a$ (II)

restricted, the following relation is checked for consistency: $t_{j,0}^{s,a} \leq t_{j,1}^{s,a}$. Otherwise, an unfeasibility has been found so that the search algorithm is allowed to backtrack safely.

Let us consider a couple of links $s - a$ and $s' - a$ ordered in such a way that $\left(t_{j,0}^{s,a} < t_{j',0}^{s',a}\right)$ *or* $\left(t_{j,0}^{s,a} = t_{j',0}^{s',a}\ \textit{and}\ t_{j,1}^{s,a} < t_{j',1}^{s',a}\right)$.

Figure 1 shows the opportunity windows as derived by the association rule. From the figure, it becomes clear that link $s' - a$ can never start within the interval $[t_{j',0}^{s',a}, t_{j,0}^{s,a} + t^C)$ (represented as a solid interval) unless the intersection between both events is at least as large as t^C.

Figure 2 shows a plausible case where $s' - a$ can happen within the aforementioned interval —the solid segments actually represent the interval within which the link operations take place.

From these observations, the following **forward propagation rule** is proposed:

$$t_{j',0}^{s',a} = max\left(t_{j',0}^{s',a}, t_{j,0}^{s,a} + t^C\right)\ \textit{iff}\ t^C \geq t_{j,1}^{s,a} - t_{j',0}^{s',a} \quad (7)$$

In a general sense, it can be easily demonstrated that link $s - a$ can only affect other links $s' - a$ such that the opportunity window of the latter starts within the interval $[t_{j,1}^{s,a} - t^C, t_{j,1}^{s,a} + t^C]$: if the start time of the opportunity window of $s' - a$ starts either before or later than the aforementioned interval, link $s' - a$ could commence at the beginning of its opportunity window and link $s - a$ could commence at the end of its opportunity window. Note that a feasible solution exists so that no intersection happens.

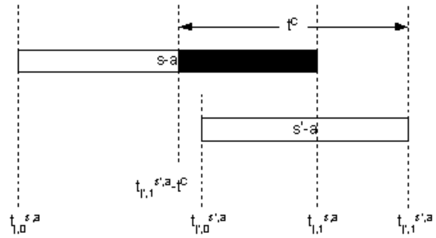

Figure 3: Intersection of the opportunity windows $s - a$ and $s' - a$ (III)

Simultaneously, $s' - a$ can trigger the propagation rule to other links if and only if it has been further restricted by the forward propagation rule, i.e, if and only if: $t_{j',0}^{s',a} < t_{j,0}^{s,a} + t^C$. From the previous paragraph, it follows also that $t_{j',0}^{s',a} > t_{j,1}^{s,a} - t^C$ —otherwise, its opportunity window would have not been affected. Putting both inequalities together it turns out:

$$\begin{aligned} t_{j,1}^{s,a} - t^C &< t_{j,0}^{s,a} + t^C \\ t^C &> \frac{1}{2}\left(t_{j,1}^{s,a} - t_{j,0}^{s,a}\right) \end{aligned}$$

This expression yields the critical value of t^C for which a new link $s - a$ can affect the domain of the legal avlues of the start time of other opportunity windows $s' - a$.

2.9 Backward propagation rule

Let us now consider the case shown in figure 3. As it can be seen, the link $s - a$ can never happen within the interval $[t_{j',1}^{s',a} - t^C, t_{j,1}^{s,a}]$ unless the intersection between them is at least as large as t^C. From these observations, the following **backward propagation rule** is proposed:

$$t_{j,1}^{s,a} = min\left(t_{j,1}^{s,a}, t_{j',1}^{s',a} - t^C\right)\ \textit{iff}\ t^C \geq t_{j,1}^{s,a} - t_{j',0}^{s',a} \quad (8)$$

In case the backward propagation rule modifies $t_{j,1}^{s,a}$, its new value could also affect other links with the same antenna a. It can be demonstrated that link $s' - a$ can only affect the opportunity windows of other links $s - a$ if and only if: $t_{j,1}^{s,a} \in \left[t_{j',0}^{s',a} - t^C, t_{j',0}^{s',a} + t^C\right]$; otherwise, both opportunity windows can safely coexist: $s' - a$ can start at the beginning of its opportunity window and room is left for $s - a$ as shown in figure 4.

Thus, the stop time of the opportunity window of a link $s - a$ can be affected by applying the backward propagation rule from a link $s' - a$ if and only if $t_{j,1}^{s,a} > t_{j',1}^{s',a} - t^C$. Likewise, link $s' - a$ is likely to affect other links $s - a$ if and only if $t_{j',0}^{s',a} + t^C > t_{j,1}^{s,a}$. Putting both inequalities together:

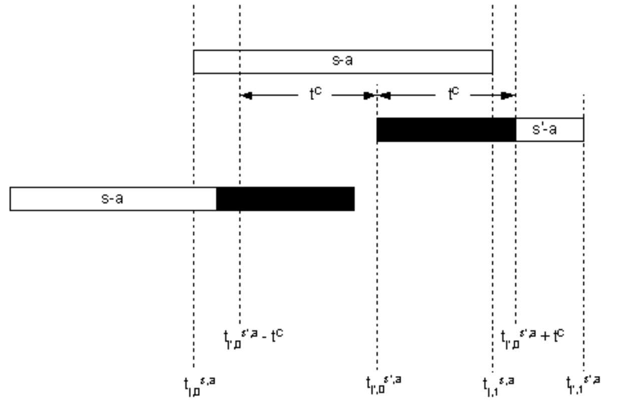

Figure 4: Triggering the backward propagation rule to other links

$$
\begin{aligned}
t_{j',0}^{s',a} + t^C &> t_{j',1}^{s',a} - t^C \\
t^C &> \frac{1}{2}\left(t_{j',1}^{s',a} - t_{j',0}^{s',a}\right)
\end{aligned}
$$

which is the same critical value of t^C derived when considering the forward propagation rule.

It shall be taken into account that both propagation rules are somewhat related to each other:

- The forward propagation rule is likely to modify the start time of a link $s' - a$ such that the stop time of other link $s'' - a$ might fall within the interval $[t_{j',0}^{s',a} - t^C, t_{j',0}^{s',a} + t^C]$ so that the backward propagation rule can be applied to it.
- The backward propagation rule is likely to modify the stop time of a link $s' - a$ such that the start time of other link $s'' - a$ might fall within the interval $[t_{j,1}^{s,a} - t^C, t_{j,1}^{s,a} + t^C]$ so that the forward propagation rule can be applied to it.

Therefore, whenever a new link $s - a$ serves for constraining the bounds of other links $s' - a$, the forward and backward propagation rule shall also be applied from the latter. Thus, ensuring the global consistency of the opportunity windows. It shall not be expected, however, that these changes will be propagated throughout the whole makespan. Instead, they should induce local changes to the schedule.

2.10 Vertical feasibility rule

Both, the forward and backward propagation rules are intended to ensure the availability of the same resource for all links. However, they do not serve to check whether resources are overused. In this section, a feasibility rule is introduced with this aim.

Figure 5 shows a plausible case for the following parameters: $t^C = 5, t_{min}^S = 10, t_{max}^S = 20$. The upper half shows

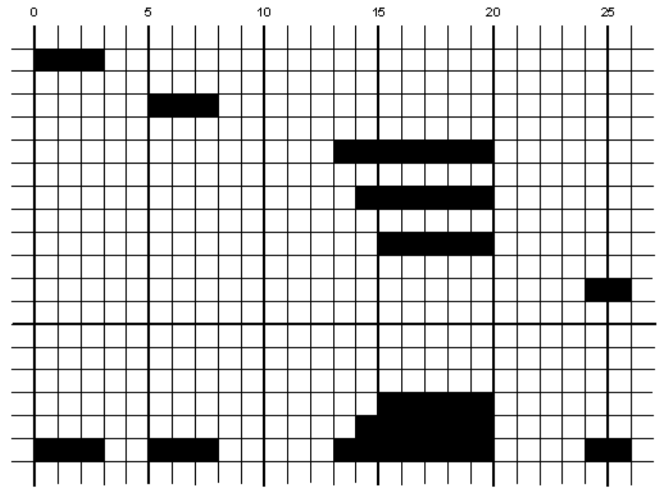

Figure 5: Vertical feasibility rule

different allocations of the same antenna to various satellites, in the form of opportunity windows as derived by applying the forward and backward propagation rules to the events generated with the association rule. The lower half of the figure shows a histogram of the assignments during all the makespan $[0, 26]$.

It can be easily checked that all events meet the forward and backward propagation rules. However, this plan is unfeasible: there are three links in the interval $[13, 20]$. Considering a contact time equal to 5 units, the first one could start at $t = 13$, the second at $t = 18$ so that the last one should start at $t = 23$, outside the interval.

Therefore, the following **vertical feasibility rule** is also introduced: assuming a contact plan is feasible, a new link $s - a$ is feasible if and only if:

$$\tau_1 - \tau_0 \geq \eta t^C \tag{9}$$

where $[\tau_0, \tau_1]$ are the bounds of the timeline of all events intersecting with the new links $s - a$ and η is the number of events within that interval —without considering the new one $s - a$.

In the case shown in figure 5, $[\tau_0, \tau_1] = [13, 20]$ and $\eta = 2$ —before considering the insertion of the opportunity window $[15, 20]$. Therefore, $20 - 13 \not\geq 2 \times 5 = 10$, so that the aforementioned event cannot be included. It is very easy to verify that instead of $[15, 20]$ a new link in the interval $[15, 23]$ is feasible, since in this case $[\tau_0, \tau_1] = [13, 23]$ so that $23 - 13 \geq 2 \times 5 = 10$.

It shall be noted that the vertical feasibility rule does not trigger any constraint propagation rule. It just checks whether it is feasible to add a new link to a contact plan which is known to be feasible.

2.11 Horizontal propagation rule

There is, however, a last outstanding issue: links are added to a contact plan which is checked for feasibility after updating the bounds of all events by propagating the forward and backward propagation rules. Thus, the fact that the start time of two successive contacts (with antenna a and next with an arbitrary antenna a') of the same satellite fall within the interval $[t^S_{min}, t^S_{max}]$ might not be true.

Therefore, the following **horizontal propagation rule** is also proposed: check that $t^S_{min} \leq t^{s,a'}_{j,0} - t^{s,a}_{j-1,0} \leq t^S_{max}$ for every satellite s.

Two different cases might appear when the horizontal propagation rule is not met:

1. $t^{s,a'}_{j,0} - t^{s,a}_{j-1,0} > t^S_{max}$: shrink the previous opportunity window by making: $t^{s,a}_{j-1,0} = t^{s,a'}_{j,0} - t^S_{max}$.
2. $t^{s,a'}_{j,0} - t^{s,a}_{j-1,0} < t^S_{min}$: shrink the next opportunity window by making $t^{s,a'}_{j,0} = t^{s,a}_{j-1,0} + t^S_{min}$.

Obviously, in case any of these cases is applied, the forward and backward propagation rules shall be applied to the event that was updated since that change might affect other links.

2.12 Bound effects

An important issue to take into account is that the patterns generated by GAST for a makespan shall be repeatable. This requirement imposes special constraints on the first and last contacts of every satellite within the makespan. More specifically:

- Selection of opportunity windows:

$$\begin{array}{rcl} t^{s,a}_0 & \in & \left[T_0, T_0 + t^S_{max}\right] \\ t^{s,a}_N & \in & \left[T_1 - t^S_{max}, T_1\right] \end{array}$$

  where $[T_0, T_1]$ is the makespan and N stands for the last contact of satellite s.

- Association of satellites with antennas:

$$\begin{array}{rcl} t^{s,a}_{0,0} & = & max\left(T_0, t^{s,a}_{0,0}\right) \\ t^{s,a}_{0,1} & = & min\left(T_0 + t^S_{max}, t^{s,a}_{0,1} - t^C\right) \end{array}$$

- Stop condition for scheduling satellite s: satellite s shall not be scheduled anymore if and only if the following conditions are satisfied:

$$t^{s,a}_{N,0} + t^S_{min} \leq T_1 + t^{s,a}_{0,0} - T_0 \leq t^{s,a}_{N,0} + t^S_{max} \tag{10}$$

3 Conclusions

From the previous discussion, the following conclusions can be drawn:

- Only one single-agent search algorithm is used to optimize the assignment of antennas to satellites during the phase of the Galileo constellation so that it can be repeated during the mission lifetime. Various constraint satisfaction rules have been derived to ensure the availability of resources at the time they are never overused.
- The depth of the search tree, d, is bounded:

$$N_s \frac{T_1 - T_0}{t^S_{max}} \leq d \leq N_s \frac{T_1 - T_0}{t^S_{min}} \tag{11}$$

  and this is the reason why the algorithm DFBnB is the best choice.
- The same search procedure solves both problems simultaneously: scheduling and resource analysis. This commitment is met by delaying the consideration of using non-built resources (antennas with priority 0) until it has been proved that no solution exists with the current resources. Simultaneously, in case a schedule is found with the current resources, it is immediately returned without suggesting the operator to make use of the possible locations specified by her.

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