Static and Dynamic Models of Observation Toward Earth by Agile Satellite Coverage

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Abstract

In mission planning of agile satellite, it is needed to decompose area targets into small pieces and compute visible time windows for sub-area targets. Therefore, it is expected to compute geodetic location of ground target observed or to compute observing time by satellite with some certain sensing actions, such as slew-looking. This paper presents static and dynamic models of observing ground targets by agile satellite for above two problems respectively. The static model is to compute the geodetic location coordinates at which boresight of sensor with a certain angle (yaw, pitch and roll angle) is pointing. Furthermore a reverse model, to be used in dynamic model, is derived from the static model to compute subsatellite point (SSP) from which Agile Satellite can observe the target, if target location is known. Then, the dynamic model is designed to determine the feasible observing SSPs and the pointing angle of satellite pointing to the known target. The observing SSPs corresponds to feasible observing time, and the pointing angle is sensing action for slew-looking. The dynamic model use the reverse model developed from the static model to compute pointing angle according to a series of SSPs, and then get intersection point of pointing angle and capability of agile satellite. With dynamic models, it is proved that the feasible time windows for one ground point are not always continuous. Finally experiments are presented and compared with results by STK (Satellite Tool Kit) to verify proposed models.

Introduction

Equipped with imaging or radar instruments, with gyroscopic actuators with which the satellites are able to move freely around their inertial center along the three axes (yaw, pitch and roll angle) (Beaumet 2006), Agile Earth Observing Satellites (AEOS) are placed on heliosynchronous, low altitude, circular orbits around the Earth (Beaumet and Verfaillie 2007; Lematre et al. 2002).

AEOS system will be an important direction for AEOS's agility and high return. For a non-agile satellite, observable scope is just a line along roll axes, which means observation is only possible when the satellite flies over the target, so the realization window of an observation is fixed. For an agile satellite (see Figure 1), observable scope is an area along roll and pitch axes, which means observation is possible

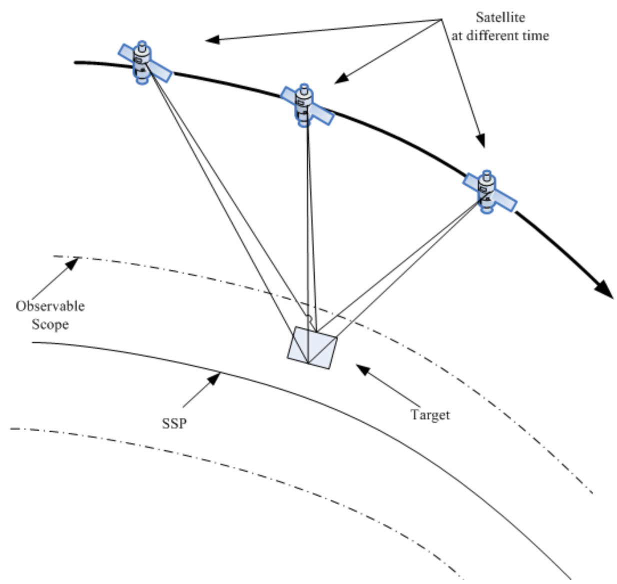

Figure 1: Time Window of agile satellite observing toward earth

before, during, or after AEOS flying over the target ground area. Consequently, much more opportunities are available for taking image of specific areas defined by users, leading to a potentially better efficiency.

The purpose of satellite observation toward Earth is to observe or communicate with points on Earth surface. Thus the spatial and temporal relationship between satellites and targets on Earth should be accurately confirmed. Regardless of imaging sensor or radar, the "visible" target should be under the coverage of spherical cap formed by satellite. The term "coverage" means that target can be imaged or detected directly by a line of sight at the center of aperture from satellite-borne imaging or radar equipment (Xu et al. 2010).

Generally speaking, planning and scheduling of satellite means select and scheduling observation resources and time among visible time windows in which targets can be observed under satellites' coverage (Lematre et al. 2002). Then there are usually two problems arising: first, what geodetic position (latitude and longitude) of a point on Earth surface can be observed by slew-looking when satellite orbits and

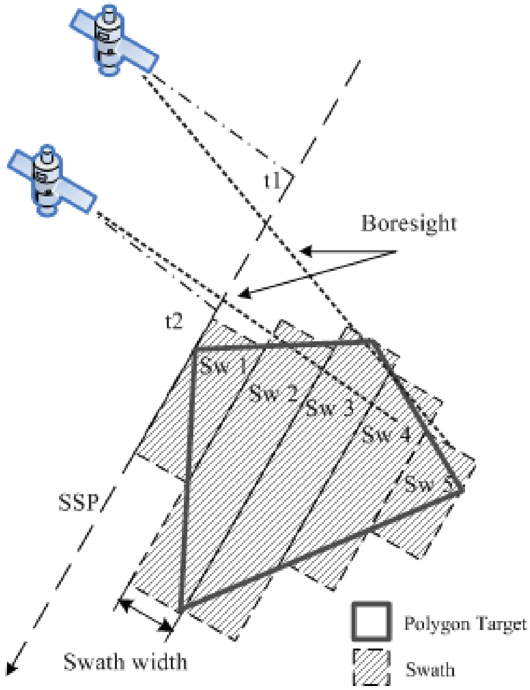

Figure 2: Polygon target covered by satellite swaths

subsatellite points (SSP) are known; second, when target is able to be observed and what angle of satellite is while position of target and satellite orbits are known. The former problem is not involved with Earth rotation, so it just needs a static model that gives the position at a certain pointing angle. However, the latter one needs to consider Earth rotation, So it has to need a dynamic model to deal with time and get the feasible SSPs and slewing angle simultaneously.

The models are necessary for mission planning and scheduling of agile satellite, while not for non-agile satellite before. For non-agile satellite, both the visible time window and slewing angle are fixed, so there parameters can be known before mission planning and scheduling. However, For agile satellite, when we select a specific time within visible time windows, in which there are infinite number of slewing angles according to continuous time, the slewing angle has directly influence on slewing action (slewing time and power consumption), which may also have influence on selection of last observation. This problem is **time dependence** (Lian and Xing 2011). when we decompose area targets into small pieces as non-agile satellite, we just consider the rolling ability at t2, then we will miss some pieces as Sw 5 in Figure 2, which is able to be observed by satellite at t1, not in t2. This problem is **task missing**. Thus, we must compute the position at a certain angle, the visible time window for an target and the pointing angle for an target at a certain time in the period of mission planning and scheduling of agile satellite.

If above two problems are settled, the spatial and temporal relationship between satellites and targets can be determined. In practice, STK (Satellite Tool Kit) can compute visible time windows for point target or area target e.g. polygon target. Because of limited satellite FOV (field of view), once observation may only cover part of area target. Whereas in visible time window computation, STK merely consider whether area target is visible, not whether area target can be totally covered. Therefore, area target should be firstly decomposed with sub-area targets according to FOV, and then compute visible time windows for these sub-area targets covered by satellite swaths, shown as Figure 2.

By this way, both visibility and total coverage can be simultaneously guaranteed. This can not be expediently offered by STK. Thus, we use static model to compute edge of swaths and then use dynamic model to compute feasible observing time window at intersections of swaths' edges and polygon target as target's visible time window. This is the motivation of this paper.

Xu (Xu et al. 2010) had develop the static and dynamic models for non-agile satellite with only one direction slewing ability, but not suitable for agile satellite with three dimension's freedom. Agility means significant flexibility in assigning start times within observation windows (Beaumet, Verfaillie, and Charmeau 2008; Lian and Xing 2011).

This paper is summarized as follows. First, notations are defined and the background foundation of Earth satellite orbital mechanics is provided; then we present the static model for the position at a certain pointing angle and the dynamic model for the feasible SSPs and slew-looking angle at a certain target and satellite orbit; last, experimental results are provided in order to show that the proposed approach is comfortable for AEOS mission planning.

Notations and Preliminaries

Several variables used in this paper are listed as follow.

lat: latitude of SSP, $lat \in [-\pi/2, \pi/2]$

lon: longitude of SSP, $lon \in [-\pi, \pi]$

$latt$: latitude of target, $lat_T \in [-\pi/2, \pi/2]$

$lont$: longitude of target, $lon_T \in [-\pi, \pi]$

inc: inclination of satellite orbit, $inc \in [0, \pi/2]$

γ: roll angle, i.e. angle of rotation about the X axis of the reference coordinate system which is Local Vertical Local Horizontal (LVLH), also known as the Gauss frame. X axis is outward along the radial (local vertical), Y is perpendicular to X in the orbit plane in the direction of motion (local horizontal), and Z is along the orbit normal (AGI 2005), $\gamma \in [-\pi/2, \pi/2]$

β: : pitch angle, i.e. angle of rotation about the Y axis of the reference coordinate system, $\beta \in [-\pi/2, \pi/2]$

α : yaw angle, i.e. angle of rotation about the Z axis of the reference coordinate system, $\alpha \in [-\pi/2, \pi/2]$

$R_x(\gamma)$: the rotation matrix of roll angle γ, as defined in (LaValle 2006)

$R_y(\beta)$: the rotation matrix of pitch angle β

$R_z(\alpha)$: the rotation matrix of yaw angle α

h: altitude of satellite

R: radius of Earth (6370 km)

Rotation matrix relation depends on the rotation sequence of yaw, roll and pitch (LaValle 2006), Without loss of generality, set $yaw \rightarrow pitch \rightarrow roll$ as rotation order in this

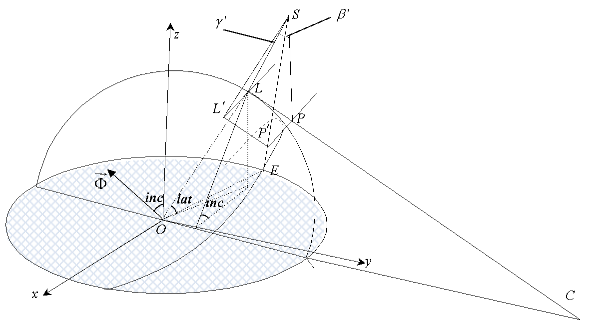

Figure 3: Geometrical illustration in slew-looking model

paper. Then, we get the boresight vector which is relative to vector (0,0,1) from the YRP angle.

$$Boresight_{Vector} = R_x(\gamma) * R_y(\beta) * R_z(\alpha) * (0,0,1) \quad (1)$$

Static Model

When satellite is above a certain SSP, how to calculate the ground location coordinates at which boresight of sensor with certain angle is pointing. This problems belongs to ”static” one, because given a certain satellite’s position it need not consider time which complicates observing situation involving the earth rotation. Without loss of generality, Figure 3 illustrates transient geometry in slew-looking when satellite with $inc \geq 90^0$ is descending or with $inc < 90^0$ ascending above northern hemisphere.

In this situation, satellite is located at S whose SSP is just at L. In Figure 3, the hatched plane is equatorial plane. E is the ground point after a roll rotation α and then a pitch rotation β . O-xyz is Earth reference frame, while O is the earth center. We can get the coordinate of point L from SSP value (lat, lon) referring to reference frame WGS-84 (World Geodetic System 1984), the geodetic coordinates are converted to XYZ coordinates by equations (Xiaoning and Wei 2003):

$$\begin{cases} x = N \cos lat \cos lon \\ y = N \cos lat \sin lon \\ z = N(1-e^2) \sin lat \end{cases} \quad (2)$$

curvature radius in prime vertical and eccentricity is respectively defined as follow:

$N = a \Big/ \sqrt{1 - e^2 \sin^2 lat}$, $e = \sqrt{a^2 - b^2} \big/ a$

where α is semi-major axis and β is short half axis.

Add plane $LPP'L'$ vertical to $\overrightarrow{OS}$ whose intersection with $\overrightarrow{SE}$ is p'. P is the intersection of plane $LPP'L'$ and $\overrightarrow{SP}$ which is the vector after pitch rotation β', while L’ is after roll rotation γ'. C is the intersection of $\overrightarrow{LP}$ and equinoctial plane. Set $\overrightarrow{\Phi}(a,b,c)$ as unit vector vertical to ΔLOC. Then,

$$\cos(inc) = (\overrightarrow{\Phi}, \overrightarrow{oz}) \Big/ (\left|\overrightarrow{\Phi}\right| * |\overrightarrow{oz}|) \quad (3)$$

$$(\overrightarrow{\Phi}, \overrightarrow{OS}) = 0 \quad (4)$$

So as to the parameter of $\overrightarrow{\Phi}(a,b,c)$ is

$$c = \cos(inc) \quad (5)$$

$$a = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (6)$$

Where, $A = x_S^2 + y_S^2, B = 2x_S z_S c, C = z_S^2 c^2 + (c^2 - 1) y_S^2$

$$b = -(a x_S + c z_S)/y_S \quad (7)$$

The satellite orbit plane can be described as

$$ax + by + cz = 0 \quad (8)$$

It is known that the length from ground point to earth center is about R(earth is a ellipsoid), i.e.

$$\begin{aligned} R &\cong \left|\overrightarrow{OE}\right| = \left|\overrightarrow{OS} + \lambda * \overrightarrow{SP'}\right| \\ &= \left|(1-\lambda) * \overrightarrow{OS} + \lambda * (\overrightarrow{OL} + \overrightarrow{LP} + h * \tan(\gamma') * \overrightarrow{\Phi})\right| \end{aligned} \quad (9)$$

It is known that $\overrightarrow{LP} \perp \Delta SLL'$, then

$$\begin{vmatrix} i & j & k \\ x_L - x_{L'} & y_L - y_{L'} & z_L - z_{L'} \\ x_S - x_{L'} & y_S - y_{L'} & z_S - z_{L'} \end{vmatrix} = x_{LP} i + y_{LP} j + z_{LP} k \quad (10)$$

And,

$$\overrightarrow{OL'} = \overrightarrow{OL} + sign(\gamma') * |LL'| * \overrightarrow{\Phi} \quad (11)$$

Combine (10),(11), we obtain:

$$\begin{cases} x_{LP} = ((S^\# x_L + \cos(inc) z_L) z_L / y_L + \cos(inc) * y_L) \\ \qquad * \tan(\gamma') * h^2 / R \\ y_{LP} = \tan(\gamma') * h^2 / R * (\cos(inc) * x_L - S^\# * z_L) \\ z_{LP} = (-\tan(\gamma') * (S^\# x_L + \cos(inc) z_L) / y_L - y_L / R) \\ \qquad * h^2 / R * x_L \end{cases} \quad (12)$$

where,

$$\begin{aligned} S^\# = &(\pm y_L \sqrt{-z_L^2 \cos^2(inc) + (x_L^2 + y_L^2)(\cos^2(inc) - 1)} \\ &- x_L z_L \cos(inc)) / (x_L^2 + y_L^2) \end{aligned}$$

So, combine (5), (6), (7), (9) and(12), we obtain λ, then,

$$\overrightarrow{OE} = (1-\lambda) * \overrightarrow{OS} + \lambda * (\overrightarrow{OL} + \overrightarrow{LP} + h * \tan(\gamma') * \overrightarrow{\Phi})$$

In addition, we can get the γ' and β' from (1):

$$\gamma' = \tan^{-1}(\tan(\gamma)/\cos(\beta)) \quad (13)$$

$$\beta' = \beta \quad (14)$$

In this way, we have easily got the groud point E.

Dynamic Model

Now we consider another problem that when target is able to be observed and what angle of satellite is while position of target and satellite orbits are known. It is equal to find out the SSPs above which the boresight of sensor with a certain angle is able to point at the target. This problem is dynamic since we have to determine an appropriate SSP boundary from a series of SSPs that imply a time sequence and can be calculated by satellite orbit prediction. Once time is under consideration, the observation situation will be complicated by the effect of satellite motion and the earth rotation.

In software STK, from AER report we can get a record of date/time, azimuth, elevation and range values between satellite (sensor) and target for the interval start and end times and for each ephemeris point available (AGI 2005). However, the adopted model in STK is not explicit and we have to convert the result into another style for area target decomposition and visible time window computation which may be quite inconvenient and inefficient in our applications. Thus a dynamic model is needed. In this section, a model is presented to calculate SSPs (time window) and angle for above dynamic problem.

Angle Model

We will not use orbit determination model involved two-body problem (Chiyangcabut 1999; Washburn 1997), because it is not necessary in our application. The main idea is that develop the reverse model of the static model presented to compute slewing angle of satellite with target location and SSP.

In angle model, it is known that target location and SSP at a time, the problem is how to compute the slewing angle of satellite.

$$\begin{aligned}\overrightarrow{LP'} &= \overrightarrow{LS} + \overrightarrow{SP'} \\ &= \overrightarrow{OL}h/R - hR\overrightarrow{SE}\Big/(\overrightarrow{OL}, \overrightarrow{SE})\end{aligned} \tag{15}$$

Where, $\overrightarrow{SE} = \overrightarrow{OE} - \overrightarrow{OS} = \overrightarrow{OE} - (R+h)\overrightarrow{OL}\Big/R$ We can obtain γ' from above:

$$tan(\gamma') = \left|\overrightarrow{LL'}\right|\Big/\left|\overrightarrow{SL}\right| = (\overrightarrow{LP'}, \overrightarrow{\Phi})\Big/h \tag{16}$$

Similarly, obtain β':

$$tan(\beta') = \left|\overrightarrow{LP}\right|\Big/\left|\overrightarrow{SL}\right| = \sqrt{\left|\overrightarrow{LP'}\right|^2 - (\overrightarrow{LP'}, \overrightarrow{\Phi})^2}\Bigg/h \tag{17}$$

Reverse the (13) and (14), combine (16) and (17), we can obtain the roll rotation angle and pitch rotation(set yaw as 0):

$$\beta = \arctan(\sqrt{\left|\overrightarrow{LP'}\right|^2 - (\overrightarrow{LP'}, \overrightarrow{\Phi})^2}\Bigg/h) \tag{18}$$

$$\gamma = \arctan((\overrightarrow{LP'}, \overrightarrow{\Phi})\Big/h) \tag{19}$$

Where $\overrightarrow{LP'}$ is derivee from (15).

Time Window Model

Definition 1 *If boresight of satellite (sensor) with a slewing angle is able to point at a target on the earth when it is above a certain SSP, this SSP is called as a access SSP of the target.*

The access SSP is influenced by the distance from target to SSPs and the ability of Satellite (slewing angle limen, such as maximum angle of roll rotation or pitch rotation), which will be proved by the experimentation in part V.

Definition 2 *For a certain target, time window (Lian and Xing 2011) of satellite access to the target consist of access SSPs in a single satellite orbit.*

Time window is an important constraint in mission planning and scheduling process, so the problem is to get the precise SSP boundary according the maximum ability in time window model. We will not use orbit determination model involved two-body problem, because it is not necessary in our application. The main idea is that develop the reverse model of the static model presented to compute angles with target location and SSPs, compute virtual angles (supposing satellite is always pointing at the target while locates at SSPs) according to a series of SSPs, and then get intersection point of virtual angles and maximum angle limen. Dynamic model's procedure is given as follow:

Step 1 *Search the closest SSP as dynamic model of non-angle satellite in (Xu et al. 2010).*

Step 2 *Select a certain part of SSPs within half one circle for reducing computation.*

Because the ability of satellite restric the most of SSPs will not feasible. Pruning SSPs will reduce computation for next step of determining the intersection point. Get the maximum and minimum of SSPs' latitudes, denoted as $[lat_{\min}, lat_{\max}]$. Then select a partition from SSPs, s.t. selected SSPs' latitude are within $[lat_{\min}, lat_{\max}]$.

Step 3 *Compute virtual angles according to a series of SSPs.*

Given target location (lat_t, lon_t), mean value of satellite altitude h and its SSP, use reverse process of static model to compute the called slewing angle as in angle model.

Step 4 *search precise boundary SSPs by determining the intersection point of virtual angles and maximum angle limen.*

$\{ssp_1, ssp_2 \cdots ssp_m\}$ and $\{\beta_1, \beta_2 \cdots \beta_n\}$ is respectively denoted as location and angle with which satellite could point at target at the location by Step2 and Step3. Initiate ssp0 as closest ssp by Step1, β_0 as the initial angle respectly. Searching procedure (illustrated by Figure 4) is as follow:

Above procedure gives the SSP which is the boundary of access SSP and unaccess SSP. the roll angle is similar to above process. Then, select the boundary SSPs to construct the time window in which both Roll and Pitch are feasible. Thus, search is complete.

Experimentation

We simulate satellites with orbit inclination 0^0, 45^0 and 90^0 and with one sensor in STK. In Figure 5, the black lines drawn on the map by STK are observable scope borderlines

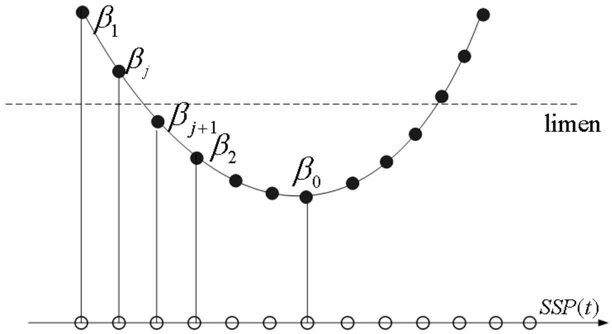

Figure 4: Search two closest points

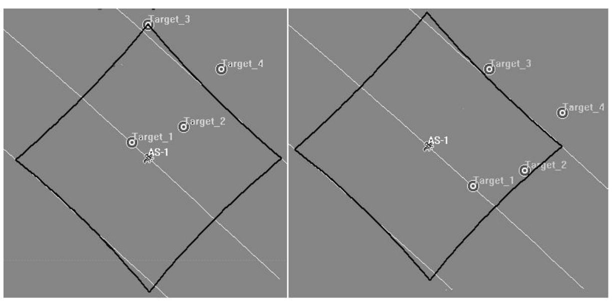

Figure 5: the relation of targets and observable scope of satellite

at two time. There are four typical targets in our experimentation, according to the distance from Target to SSPs related to angle limen.

Results

Table 1 shows the differents between the points after slew-looking by static model and STK about 242 random SSP and 25600 angles, i.e. 6195200. Table 2 shows that angles which are computed by angle model of dynamic model are compared by the real angles in STK. Simulation results indicate two models have efficient precision. Moreover, static and dynamic model needs tiny computation consumptions. 6 shows the time window of four typical target, which computed by time window model of dynamic model.

Geometrical Analysis

In Figure 6, (1), (2), (3), (4) are the time window of $Target\_1$, $Target\_2$, $Target\_3$ and $Target\_4$ shown in Figure 5.

For $Target\_1$, Figure 6 (1) shows that pitch angle first reduced, then rised up, roll angle keeped very low, small waved. Time window is determined by pitch. The reason to explain it is the distance from target to SSPs. Clearly, $Target\_1$ is nearly on SSPs.

For $Target\_2$, Figure 6 (2) shows that pitch angle is similar to $Target\_1$, but roll angle is in oppsite of pitch, first rised up, then reduced. Time window is determined by pitch, but roll angle nearly reached limen. It is clearly because the target is too far from SSPs that satellite must give much roll to point at the target.

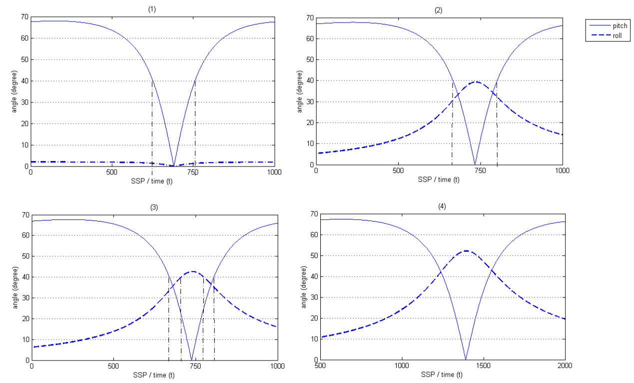

Figure 6: The time window of four typical target

For $Target\_3$, Figure 6 (3) shows that roll angle is high enough that Time window is determined by both roll and pitch. In this situation, time window is not continual.

For $Target\_4$, Figure 6 (4) shows that roll angle is too high that Time window is none. In this situation, roll is the main factor for access.

Conclusion

Based on the geometrical relation between satellite and ground target, the proposed agile static model can compute observed point location under three freedom degree slew-looking, and dynamic model can present angles of pointing and time window of target. By simulation, both models have adequate precision for satellite (sensor) mission planning but are quite low computationally demanding.

After appearance of static and dynamic model of agile satellites , also arising a problem: how to make decision in mission planning, we will develop dynamic decision support involved dynamic area target division for agile satellite in the future work.

Acknowledgments

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Table 1: Comparision of ground points computed by Static Model and STK about 242 random points (units: 10^{-5} degree)

Inclination	Roll Limen	Pitch Limen	Mean Error of Latitudes	Standard deviation Error of Latitudes	Mean Error of Longitudes	Standard deviation Error of Longitudes
0	[-40,40]	[-40,40]	11.861	13.621	5.774	8.298
45	[-40,40]	[-40,40]	296.436	16.5303	22.917	29.408
90	[-40,40]	[-40,40]	460.436	288.735	100.551	134.105

Table 2: Comparision of angle computed by Dynamic Model and STK about random SSPs (units: degree)

Inclination	Roll Limen	Pitch Limen	Mean Error of Pitch	Standard deviation Error of Pitch	Mean Error of Roll	Standard deviation Error of Roll
0	[-40,40]	[-40,40]	0.00046	0.00051	0.00049	0.00046
45	[-40,40]	[-40,40]	0.22709	0.12984	0.22254	0.11411
90	[-40,40]	[-40,40]	0.00049	0.00028	0.46485	0.26135

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