A TRAJECTORY AND FORCE CONTROL OF A MANIPULATOR WITH ELASTIC LINKS

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ABSTRACT

When the velocities of the motion required for a manipulator become faster, and the values of forces acting on a subject are large, the elastic deformations of the manipulator cannot be ignored in the trajectory and force control. To develop a method of the trajectory and force control of a flexible manipulator is an important problem. In this paper, a trajectory and force control of a flexible manipulator based on inverse kinematics and inverse dynamics is proposed. First, inverse kinematics and inverse dynamics of a flexible manipulator are investigated in detail, and then, a hierarchical controller based on inverse kinematics and inverse dynamics is proposed. The performances of the proposed controller are verified by numerical simulations.

1. INTRODUCTION

A manipulator is a mechanical system whose links are connected through translational or rotational joints. One of the tasks for a manipulator is to control forces acting on a subject along to a given trajectory. In order to establish the task, a force and trajectory control is necessary. In the space engineering, this class of technology is required for assembly of space structures on an orbit or for release and retrieval of an artificial satellite.

When the velocities of the motion required for a manipulator become faster, and the values of forces acting on a subject are large, the elastic deformations of the manipulator cannot be ignored in the trajectory and force control. A manipulator whose links has elastic deformations which cannot be ignored is called a flexible manipulator, while a manipulator composed of links whose elastic deformations can be ignored is called a rigid manipulator. It is necessary for a space manipulator to be considered as a flexible manipulator because the structural rigidity of the links becomes lower through lightening its weight. Then, to develop a method of the trajectory and force control of a flexible manipulator is an important problem in order to use a space manipulator well.

In this paper, a trajectory and force control of a flexible manipulator is proposed. Generally, one of the basic methods of motion control of a manipulator is as follows; First, by using control inputs, the state equations are linearized. And then, based on the linearized state equations, linear feedback control is adopted. A method in which the state equations are linearized by compensating the nonlinear terms with the use of the measured values of the state variables is called feedback linearization method. In this method, a feedback control is executed based on the linearized state equations. On the other hand, a method in which the state equations are linearized by inputing the control force which realizes the desired motion is called feedforward linearization method. In this method, the linearized state equations are derived around the desired values of the state variables, then a feedback control is executed based on the linearized state equations.

In the feedforward linearization method, inverse kinematics and inverse dynamics are important. When the desired motion and the desired force acting on the surface of an object through the manipulator are given, inverse kinematics is to calculate all the state variables according to the motion and the force are

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calculated, and inverse dynamics is to calculate the desired input forces or torques to realize the motion derived through inverse kinematics.

For a rigid manipulator, inverse kinematics is derived using kinematic relations between the state variables and inverse dynamics is derived by using the state equations. And then, the input forces or torques can be realized by the actuators of the manipulator.

But, there are some difficulties in inverse kinematics and inverse dynamics of a flexible manipulator: Inverse kinematics cannot be derived by only the kinematic relations between the state variables. The input forces or torques derived by inverse dynamics cannot be realized by the actuators of the manipulator either.

In this paper, a trajectory and force control of a flexible manipulator based on the feedforward linearization method. Inverse kinematics and inverse dynamics of a flexible manipulator are investigated in detail, and a hierarchical controller based on inverse kinematics and inverse dynamics is proposed.

This paper is composed as follows; First, in section 2, the model of the manipulator system dealt with in this paper is mentioned and the equations of motion are derived. In section 3, the methods of inverse kinematics and inverse dynamics are derived. In section 4, the method of design of the controller is mentioned and in section 5, the performances of the proposed controller are verified by numerical simulations.

2. FORWARD MODEL

Consider a manipulator composed of two bodies, body 1 and body 2 (FIG. 1). Body 1 is put on a base with a rotary joint (joint 1) and body 2 is connected to body 1 with a rotary joint (joint 2). Motors are installed at the rotary joints, the axes of which are perpendicular to a vertical plane. Body 1 is a rigid rod and body 2 is an elastic beam, elastic deformations of which occur in a plane perpendicular to the axis of rotation. Introduce a set of unit vectors $\{\boldsymbol{a}^{(0)}\} = \{\boldsymbol{a}_1^{(0)}, \boldsymbol{a}_2^{(0)}, \boldsymbol{a}_3^{(0)}\}$ fixed in an inertia space, the origin of which coincides with joint 1. Vector $\boldsymbol{a}_3^{(0)}$ coincides with the axis of rotation and vector $\boldsymbol{a}_2^{(0)}$ is set downward. A set of unit vectors $\{\boldsymbol{a}^{(i)}\} = \{\boldsymbol{a}_1^{(i)}, \boldsymbol{a}_2^{(i)}, \boldsymbol{a}_3^{(i)}\}$ is introduced, the origin of which coincides with joint i. Vector $\boldsymbol{a}_3^{(i)}$ coincides with the axis of rotation of joint i and vector $\boldsymbol{a}_1^{(i)}$ is set toward the axis of body i. Using a set of unit vectors $\{\boldsymbol{a}^{(i)}\}$, a column matrix is introduced.

$$[\boldsymbol{a}^{(i)}]^T = [\boldsymbol{a}_1^{(i)}, \boldsymbol{a}_2^{(i)}, \boldsymbol{a}_3^{(i)}] \tag{1}$$

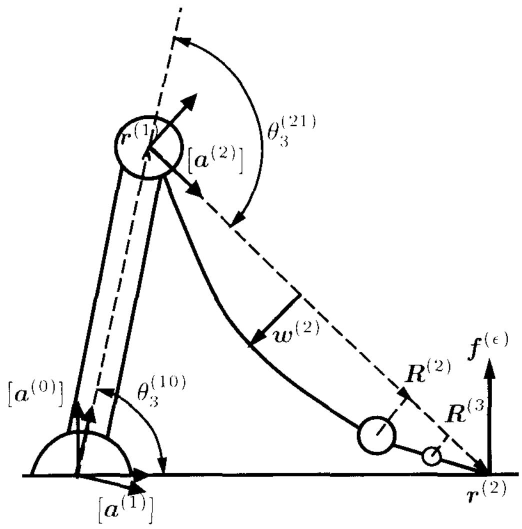

FIG. 1 Two body manipulator system

By introducing the angles of rotation from $\{\boldsymbol{a}^{(j)}\}$ to $\{\boldsymbol{a}^{(i)}\}$ about $\boldsymbol{a}_3^{(j)}$ axis as $\theta_3^{(ij)}$, transformation matrices from $\{\boldsymbol{a}^{(j)}\}$ to $\{\boldsymbol{a}^{(i)}\}$ are defined by $A^{(ij)}$.

The angular velocity vector of $\{\boldsymbol{a}^{(i)}\}$ to $\{\boldsymbol{a}^{(j)}\}$ is defined by $\boldsymbol{\omega}^{(ij)}$

$$\begin{aligned} \boldsymbol{\omega}^{(ij)} &= [\boldsymbol{a}^{(i)}]^T \omega^{(ij)} \\ \omega^{(ij)T} &= [0, 0, \dot{\theta}_3^{(ij)}] \end{aligned} \tag{2}$$

The following quantities are introduced.

$\boldsymbol{r}^{(1)} = [\boldsymbol{a}^{(1)}]^T r^{(1)}$: a distance vector from joint 1 to joint 2.

$\boldsymbol{r}^{(2)} = [\boldsymbol{a}^{(2)}]^T r^{(2)}$: a distance vector from joint 2 to the end effector.

$\boldsymbol{\rho}^{(i)} = [\boldsymbol{a}^{(i)}]^T \rho^{(i)}$: a distance vector from joint i to any position in body i.

The elastic deformation of body 2 is denoted by $\boldsymbol{w}^{(2)}$

$$\begin{aligned} \boldsymbol{w}^{(2)} &= [\boldsymbol{a}^{(2)}]^T w^{(2)} \\ w^{(2)T} &= [0,\ w_2^{(2)}(t,\ \rho_1^{(2)}),\ 0] \end{aligned} \tag{3}$$

By using the finite element method, the elastic deformation $w_2^{(2)}(t,\ \rho_1^{(2)})$ is expressed as (Appendix)

$$w_2^{(2)}(t,\ \rho_1^{(2)}) = B^{(2)}(\rho_1^{(2)})\hat{w}_2^{(2)}(t) \tag{4}$$

A distance vector $\boldsymbol{x}^{(1)}$ from joint 1 to any point in body 1 is expressed as

$$\begin{aligned} \boldsymbol{x}^{(1)} &= [\boldsymbol{a}^{(1)}]^T x^{(1)} \\ x^{(1)} &= \rho^{(1)} \end{aligned}$$

and the velocity vector $\boldsymbol{v}^{(1)}$ is expressed as

$$\begin{aligned} \boldsymbol{v}^{(1)} &= [\boldsymbol{a}^{(1)}]^T v^{(1)} \\ v^{(1)} &= \tilde{\rho}^{(1)} \omega^{(10)} \end{aligned} \tag{5}$$

where, $\tilde{\rho}^{(1)T}$ makes a cross product in $[\boldsymbol{a}^{(1)}]$.
On the other hand, a distance vector $\boldsymbol{x}^{(2)}$ from joint 1 to any point in body 2 is expressed as

$$\begin{aligned} \boldsymbol{x}^{(2)} &= [\boldsymbol{a}^{(2)}]^T x^{(2)} \\ x^{(2)} &= A^{(21)} r^{(1)} + (\rho^{(2)} + w^{(2)}) \end{aligned}$$

and the velocity vector $\boldsymbol{v}^{(2)}$ is expressed as

$$\begin{aligned} \boldsymbol{v}^{(2)} &= [\boldsymbol{a}^{(2)}]^T v^{(2)} \\ v^{(2)} &= A^{(21)} \tilde{\rho}^{(1)} \omega^{(10)} \\ &\quad + \{\tilde{\rho}^{(2)} \omega^{(20)} + B^{(2)}(\rho_1^{(2)}) \dot{\hat{w}}^{(2)}(t)\} \end{aligned} \tag{6}$$

A state variables z of the system are set to be

$$z^T = [\theta_3^{(10)},\ \theta_3^{(21)},\ \hat{w}_2^{(2)T}] \tag{7}$$

The equation of motion for stable variables z are derived as follows; The equations for variables $\theta_3^{(10)}$ and $\theta_3^{(21)}$ are derived from the equations of the angular momenta of body 1 and body 2 about joint 1 and joint 2, respectively.

$$\begin{aligned} &\frac{d}{dt}\left\langle \tilde{\rho}_c^{(1)T} v^{(1)} \right\rangle^{(1)} + \left\langle \tilde{\rho}_c^{(1)T} \tilde{\omega}^{(10)T} v^{(1)} \right\rangle^{(1)} \\ &\quad = -\tilde{r}_c^{(1)T} f^{(1)} - (\tilde{r}^{(1)T} - \tilde{r}_c^{(1)T}) A^{(12)} f^{(2)} \\ &\qquad + \tau^{(1)} - A^{(12)} \tau^{(2)} \end{aligned} \tag{8}$$

$$\begin{aligned} &\frac{d}{dt}\left\langle \tilde{\rho}_c^{(2)T} v^{(2)} \right\rangle^{(2)} + \left\langle \tilde{\rho}_c^{(2)T} \tilde{\omega}^{(20)T} v^{(2)} \right\rangle^{(2)} \\ &\quad = -\tilde{r}_c^{(2)T} f^{(2)} - (\tilde{r}^{(2)T} - \tilde{r}_c^{(2)T}) A^{(20)} f^{(e)} + \tau^{(2)} \end{aligned} \tag{9}$$

where,

$$\begin{aligned} \rho_c^{(i)} &= \rho^{(i)} - r_c^{(i)},\ r_c^{(i)} = \frac{1}{m^{(i)}} \int \rho^{(i)} dm^{(i)} \\ < * >^{(i)} &= \int * dm^{(i)} \end{aligned}$$

$m^{(i)}$ is mass of body i. $\boldsymbol{f}^{(i)} = [\boldsymbol{a}^{(i)}]^T f^{(i)}$ and $\boldsymbol{\tau}^{(i)} = [\boldsymbol{a}^{(i)}]^T \tau^{(i)}$ are a force and a torque acting on body i at joint i, and $\boldsymbol{f}^{(e)} = [\boldsymbol{a}^{(0)}]^T f^{(e)}$ is a force acting on the surface of the object through the end effector.

The forces $f^{(1)}$ and $f^{(2)}$ are expressed as

$$\begin{aligned} f^{(1)} &= \frac{d}{dt}\left\langle v^{(1)} \right\rangle^{(1)} + \tilde{\omega}^{(10)T} \left\langle v^{(1)} \right\rangle^{(1)} \\ &\quad - m^{(1)} A^{(10)} g + A^{(12)} f^{(2)} \\ f^{(2)} &= \frac{d}{dt}\left\langle v^{(2)} \right\rangle^{(2)} + \tilde{\omega}^{(20)T} \left\langle v^{(2)} \right\rangle^{(2)} \\ &\quad - m^{(2)} A^{(20)} g + A^{(20)} f^{(e)} \end{aligned} \tag{10}$$

where g is a gravitational constant. The equations of motion for variables $\hat{w}_2^{(2)}$ are derived from the equations of elastic vibrations of body 2

$$\begin{aligned} &\frac{d}{dt}\left\langle B^{(2)T} v^{(2)} \right\rangle^{(2)} + \left\langle B^{(2)T} \tilde{\omega}^{(20)T} v^{(2)} \right\rangle^{(2)} \\ &\quad = \left\langle B^{(2)T} \right\rangle^{(2)} A^{(20)} g - K^{(2)} \hat{w}^{(2)} - D^{(2)} \dot{\hat{w}}^{(2)} \\ &\qquad - B^{(2)T}(r^{(2)}) A^{(20)} f^{(e)} + E^{(2)T} \tau_3^{(2)} \end{aligned} \tag{11}$$

where, the second and third terms in the right hand side of Eq. (11) express an elastic restoring force and a structural damping force, respectively.

$$E^{(2)} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

3. INVERSE MODEL

It is assumed that a desired trajectory $\boldsymbol{x}_d^{(e)}(t) = [\boldsymbol{a}^{(0)}]^T x_d^{(e)}$ of the end effector and a desired force $\boldsymbol{f}_d^{(e)}(t) = [\boldsymbol{a}^{(0)}]^T f_d^{(e)}$ acting on the surface of an object through the end effector are given. Inverse kinematics is to calculate the angles of rotation $\theta_{d3}^{(10)}(t)$ and $\theta_{d3}^{(21)}(t)$ and the elastic deformations $\hat{w}_{d2}^{(2)}(t)$ corresponding to the desired trajectory and the desired force. First, the distance vector $\boldsymbol{x}^{(e)}$ of the end effector from joint 1 is expressed as

$$\begin{aligned} \boldsymbol{x}^{(e)} &= [\boldsymbol{a}^{(0)}]^T x^{(e)} \\ x^{(e)} &= A^{(01)} r^{(1)} + A^{(02)} r^{(2)} \end{aligned} \tag{12}$$

Substituting the desired trajectory $x_d^{(e)}$ into Eq. (12), we obtain the equation to determine the angles of rotation $\theta_{d3}^{(10)}$, $\theta_{d3}^{(21)}$.

$$\begin{aligned} x_{d1}^{(e)} &= r^{(1)} \cos\theta_{d3}^{(10)} + r^{(2)} \cos\theta_{d3}^{(20)} \\ x_{d2}^{(e)} &= r^{(1)} \sin\theta_{d3}^{(10)} + r^{(2)} \sin\theta_{d3}^{(20)} \end{aligned} \tag{13}$$

Next, substituting the desired force $f_d^{(e)}$ and the angles of rotation $\theta_{d3}^{(10)}$, $\theta_{d3}^{(21)}$ into Eq. (11), we obtain the equation to determine the elastic deformations

$\widehat{w}^{(2)}_{d2}(t)$.

$$
\begin{aligned}
&M^{(2)}\ddot{\widehat{w}}_{d2} + D^{(2)}\ \dot{\widehat{w}}^{(2)}_{d2} + \left\{K^{(2)} - \omega^{(20)2}_{d3}M^{(2)}\right\}\ \widehat{w}^{(2)}_{d2} \\
&= \Big(-g_2\cos\theta^{(20)}_{d3} - r^{(1)}_1\omega^{(10)2}_{d3}\sin\theta^{(21)}_{d3} \\
&\qquad -r^{(1)}_1\dot{\omega}^{(10)}_{d3}\cos\theta^{(21)}_{d3}\Big) < B^{(2)T}_2 >^{(2)} \\
&\quad -\dot{\omega}^{(20)}_{d3} < B^{(2)T}_2\rho^{(2)}_1 >^{(2)} \\
&\quad +\Big\{(J^{(2)} + m^{(2)}R^{(2)2})\dot{\omega}^{(20)}_{d3} \\
&\qquad +m^{(2)}r^{(1)}R^{(2)}\cos\theta^{(21)}_{d3}\dot{\omega}^{(10)}_{d3} \\
&\qquad m^{(2)}r^{(1)}R^{(2)}\sin\theta^{(21)}_{d3}\omega^{(10)2}_{d3} \\
&\qquad +r^{(2)}(-f^{(e)}_1\sin\theta^{(20)}_{d3} + f^{(e)}_2\cos\theta^{(20)}_{d3}) \\
&\qquad +m^{(2)}R^{(2)}g_2\cos\theta^{(20)}_{d3}\Big\}\ E^{(2)T} \qquad (14)
\end{aligned}
$$

$$M^{(2)} =< B^{(2)T}_2 B^{(2)}_2 >^{(2)} - < E^{(2)T}C^{(2)} >^{(2)}$$

where, $C^{(2)} = [< \rho^{(2)}_1 B^{(2)}_2 >^{(2)},\ 0,\ \cdots,\ 0\ ]$.
Equation (14) is a set of second order ordinary differential equations and are appropriate to be formulated as initial value problems with the initial conditions

$$t = 0,\qquad \widehat{w}^{(2)}_2 = \dot{\widehat{w}}^{(2)}_2 = 0$$

However, since the coefficient matrix of $\ddot{\widehat{w}}^{(2)}_{d2}$ is not positive definite, Eq. (14) is not well posed as an initial value problem. Here, Eq. (14) is formulated as boundary value problem with the boundary conditions

$$
\begin{aligned}
t = 0 &\qquad \widehat{w}^{(2)}_2 = \alpha \\
t = t_f &\qquad \widehat{w}^{(2)}_2 = \beta
\end{aligned} \qquad (15)
$$

where, t_f is a time interval of manipulation, and α,β are values of elastic deformations in a steady state.
As boundary value problems, we can obtain stable solutions numerically, but the elastic deformations $\widehat{w}^{(2)}_{d2}$ obtained have certain velocities at the beginning of manipulation.
On the other hand, when the angle of rotation $\theta^{(10)}_{d3}(t)$ and $\theta^{(21)}_{d3}(t)$ and elastic deformations $\widehat{w}^{(2)}_{d2}(t)$ are obtained, inverse dynamics is to calculate the torques $\tau^{(1)}_d$ and $\tau^{(2)}_d$ which realize the desired motions. First, the forces $f^{(1)}_d(t)$ and $f^{(2)}_d(t)$ are calculated by Eq. (10) with the variables $\theta^{(10)}_{d3}$,$\theta^{(21)}_{d3}$ and $\widehat{w}^{(2)}_{d2}$.

$$
\begin{aligned}
f^{(2)}_d &= \frac{d}{dt}\left\langle v^{(2)}_d\right\rangle^{(2)} + \tilde{\omega}^{(20)T}_d\left\langle v^{(2)}_d\right\rangle^{(2)} \\
&\quad -m^{(2)}A^{(20)}g + A^{(20)}f^{(e)}_d \\
f^{(1)}_d &= \frac{d}{dt}\left\langle v^{(1)}_d\right\rangle^{(1)} + \tilde{\omega}^{(20)T}_d\left\langle v^{(1)}_d\right\rangle^{(1)} \\
&\quad -m^{(1)}A^{(10)}g + A^{(12)}f^{(2)}_d
\end{aligned} \qquad (16)
$$

The torques $\tau^{(1)}_d(t)$ and $\tau^{(2)}_d(t)$ are calculated by using Eqs. (8),(9),(11). But, when we calculate $\tau^{(1)}_d(t)$ and $\tau^{(2)}_d(t)$ from Eqs. (8),(9),(11), it is inconsistent. In this paper, the least square solutions are used for $\tau^{(1)}_d(t)$,$\tau^{(2)}_d(t)$.

$$
\begin{aligned}
\tau^{(2)}_d &= \frac{d}{dt}\left\langle\tilde{\rho}^{(2)T}_c v^{(2)}_c\right\rangle^{(2)} + \left\langle\tilde{\rho}^{(2)T}_c\tilde{\omega}^{(20)T}_d v^{(2)}_d\right\rangle^{(2)} \\
&\quad +\tilde{r}^{(2)T}_c f^{(2)}_d + \left(\tilde{r}^{(2)T} - \tilde{r}^{(2)T}_c\right)A^{(20)}f^{(e)}_d \\
\tau^{(1)}_d &= \frac{d}{dt}\left\langle\tilde{\rho}^{(1)T}_c v^{(1)}_c\right\rangle^{(1)} + \left\langle\tilde{\rho}^{(1)T}_c\tilde{\omega}^{(10)T}_d v^{(1)}_d\right\rangle^{(1)} \\
&\quad +\tilde{r}^{(1)T}_c f^{(1)}_d + \left(\tilde{r}^{(1)T} - \tilde{r}^{(1)T}_c\right)A^{(12)}f^{(2)}_d \\
&\quad +A^{(12)}\tau^{(2)}_d
\end{aligned} \qquad (17)
$$

4. DESIGN OF A CONTROLLER

Consider a trajectory and force control of a manipulator. A trajectory and force control is a manipulation for the end effector of the manipulator to track a certain trajectory, that is, to follow a surface of an object and also to act a desired pushing force on the surface of it during the manipulation.
The proposed controller is composed of two parts; Feedforward and feedback terms. The feedforward term is to compensate nonlinearity of the system dynamics and is calculated based on inverse kinematics and inverse dynamics; Equation (17) is used.
Equation (17) includes some errors due to two factors. One factor is that Eq. (17) includes impulsive forces at the moments of the beginning and the end of manipulation because $\dot{\widehat{w}}^{(2)}_2 \neq 0$ as the solution of Eq. (14). But when Eq. (17) is applied to the feedforward controller, the components of impulsive torques are neglected. Therefore, the motion of the manipulator have some errors at the beginning and the end of manipulation.
On the other hand, Eq. (17) is the least square solution, therefore it is not the exact solution. The errors of the solution cause some vibration modes and degrade the performances of the controller.
The feedback term compensates the errors included in the feedforward term with the model errors due to neglect of vibration modes and that caused by disturbances. A flexible manipulator become often to be a non minimum phase system. As a result, the feedback loop causes the system unstable. To pre-

vent it and to insure the robustness of the controller, a direct feedback controller is an effective one. In this paper, the feedback term of the proposed controller is designed as the direct feedback one.
The input commands to the motors at the joints are designed as follows:

$$\begin{aligned} \tau_c^{(1)} &= \tau_{d3}^{(1)} & (18)\\ &+s_1\left\{K_{DF}\dot{e}_f + K_{PF}e_f + K_{IF}\int e_f dt\right\} & (19)\\ \tau_c^{(2)} &= \tau_{d3}^{(2)} - K_w \dot{\widehat{w}}'_{2,0} & \\ &+s_2\left\{K_{DF}\dot{e}_f + K_{PF}e_f + K_{IF}\int e_f dt\right\} & (20)\\ e_f &= f_{2d}^{(e)} - f_2^{(e)} & \\ s_1 &= r^{(1)}\cos\theta_3^{(10)} + r^{(2)}\cos\theta_3^{(20)} & \\ s_2 &= r^{(1)}\cos\theta_3^{(10)} & \end{aligned}$$

Feedback gains are determined as follows; First, consider deviations of variables $f_2^{(e)}$ $\widehat{w}_{2,n}$, $\widehat{w}'_{2,n}$, and the derivatives from the nominal values.

$$\begin{aligned} \widehat{w}_{2,n} &= \widehat{w}^*_{2,n} + \Delta\widehat{w}_{2,n}\\ \dot{\widehat{w}}_{2,n} &= \dot{\widehat{w}}^*_{2,n} + \Delta\dot{\widehat{w}}_{2,n}\\ \ddot{\widehat{w}}_{2,n} &= \ddot{\widehat{w}}^*_{2,n} + \Delta\ddot{\widehat{w}}_{2,n}\\ e_f &= f_{2d}^{(e)} - f_2^{(e)} \end{aligned} \tag{21}$$

Following vector is defined.

$$X = \left[\begin{array}{cccc} \Delta\dot{\widehat{w}}^{(2)T} & \Delta\widehat{w}^{(2)T} & e_f & \int e_f dt \end{array}\right]^T \tag{22}$$

Substituting Eq. (21) into Eqs. (8), (9), (11), and linearizing them following equations are obtained.

$$H_1\dot{X} + H_2X = 0 \tag{23}$$

where,

$$H_1 = \left[\begin{array}{cccc} < B^{(2)T}B^{(2)} >^{(2)} & D'^{(2)} & aE^{(2)T} & bE^{(2)T}\\ O & I & 0 & 0\\ R & S & a & b\\ < \rho_1^{(2)}B^{(2)} >^{(2)} & 0 & e & f \end{array}\right]$$

$$H_2 = \left[\begin{array}{cccc} O & K'^{(2)} & 0 & cE^{(2)T}\\ -I & O & 0 & 0\\ 0 & d & 0 & c\\ 0 & \omega_3^{(20)} < \rho_1^{(2)}B^{(2)} >^{(2)} & 0 & g \end{array}\right]$$

$$a = -s_1K_{DF}\ , \qquad b = -s_1(1+K_{PF})$$

$$c = -s_2K_{IF}$$

$$d = -\Big[r^{(1)}\sin\theta_3^{(21)}\dot{\omega}_3^{(20)} < B >^{(2)} + r^{(1)}\omega_3^{(20)2} < \rho_1^{(2)}B >^{(2)}\Big]$$

$$e = -s_2K_{DF}\ , \qquad f = -s_2(1+K_{PF})$$

$$g = -s_2K_{IF}$$

$$R = < \rho_1^{(2)}B >^{(2)} + r^{(1)}\cos\theta^{(21)} < B^{(2)} >^{(2)}$$

$$S = -2r^{(1)}\omega^{(20)}\sin\theta^{(21)} < B^{(2)} >^{(2)}$$

$$D'^{(2)} = D + K_wE^{(2)T}E^{(2)}$$

$$K'^{(2)} = K^{(2)} - \omega_3^{(20)2} < B^{(2)T}B^{(2)} >^{(2)}$$

Based on Eq. (23), feedback gains K_{DF}, K_{PF}, K_{IF} and K_w are determined appropriately.

5. NUMERICAL SIMULATION

Here, the controller proposed in section 4 are verified numerically; The desired trajectory of the end effector $x_d^{(e)}$ and the desired force $f_d^{(e)}$ acting on the surface of the object are given and the torques $\tau_c^{(1)}$, $\tau_c^{(2)}$ which realize the desired motion are calculated on the basis of the inverse models proposed. Then, the equations of motion of the manipulator are solved numerically where the torques obtained are used as the input torques, and the force $f^{(e)}$ acting on the surface are compared with the desired force acting on the surface. The values of parameters of the manipulator are listed in TABLE 1.

TABLE 1

	Link 1	Link 2
Length [m]	0.500	0.550
Mass [kg]	8.00	0.240
Bending Stiffness[Nm^2]	-	0.480
Damping Ratio	-	5.00 E-02
Natural Frequency [Hz]		
1st mode	-	4.8004
2nd mode	-	19.8850
3rd mode	-	49.0634
4th mode	-	102.8703
5th mode	-	174.6451
6th mode	-	271.9208

Body 2 is modeled as four finite elements for the inverse models ($N = 4$). The desired trajectory of the end effector and the desired force acting on the surface are given as follows,

$$\begin{aligned} x_{d2}^{(e)} &= 0.05 \quad [\mathrm{m}]\\ f_{d2}^{(e)} &= 2.0 \quad [\mathrm{N}] \end{aligned}$$

$$x_{d1}^{(e)} = 0.7 - 0.5(-252\hat{t}^{11} + 1386\hat{t}^{10} - 3080\hat{t}^{9} + 3465\hat{t}^{8} - 1980\hat{t}^{7} + 462\hat{t}^{6}) \quad [\mathrm{m}]$$

where, $\hat{t} = t/t_f$, $t_f = 2.0$ [sec].
Figure 2 shows stick diagram of the manipulator. Figures 3 $\sim$ 6 show the forces acting on the surface of the object through the end effector.

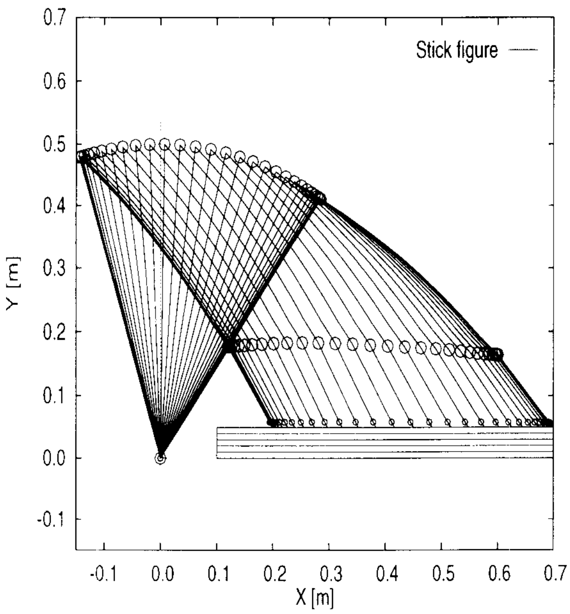

FIG. 2 Stick figure of the manipulator

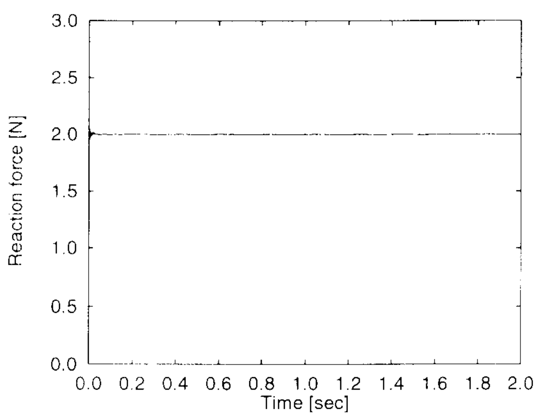

FIG. 3 The force $f^{(e)}$ acting on the surface (with initial elastic deformation velocity, feedforward only)

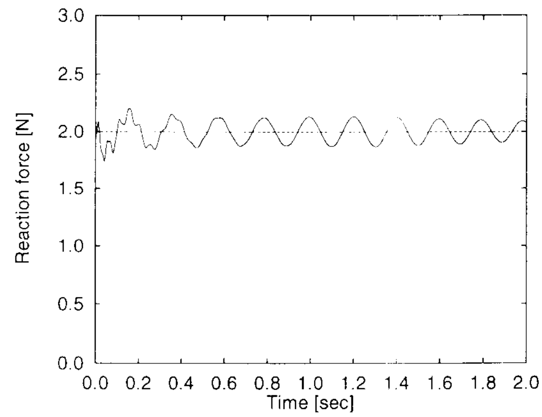

FIG. 4 The force $f^{(e)}$ acting on the surface (without initial elastic deformation velocity, feedforward only)

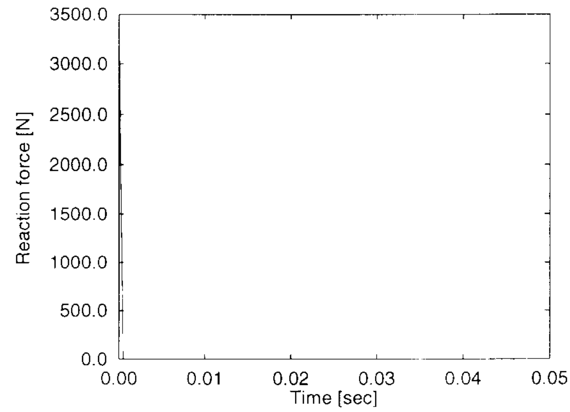

FIG. 5 The force $f^{(e)}$ acting on the surface (without initial elastic deformation velocity, feedback only)

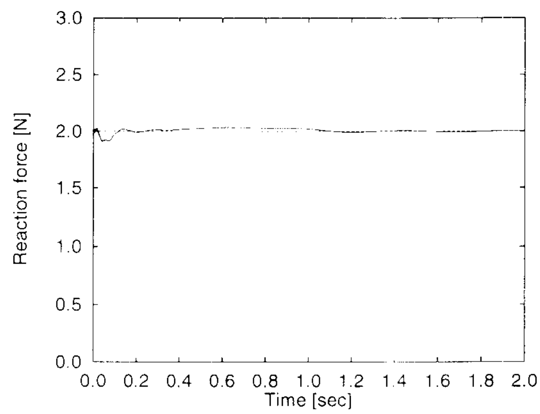

FIG. 6 The force $f^{(e)}$ acting on the surface (without initial elastic deformation velocity, proposed controller)

Figure 3 indicates the feedforward torque can control the reaction force accurately if the initial elastic deformation velocities are given. But from figure 4, we can find that if the initial elastic deformation velocities are not given(it has more reality), the feed forward controller only causes some vibrations at the beginning of the manipulation. On the other hand, from figure 5, we can find that feedback controller only cannot control motion of the manipulator. Finally, from figure 6, the proposed controller can suppress the excitation of vibration and has a good performance for a trajectory and force control of a manipulator.

6. CONCLUSION

In order to establish a trajectory and force control of a manipulator, the controllers have to generate input torque commands to the motors at the joints for the end effector to realize the desired trajectory and pushing force. In such cases, we have to consider three difficulties to design the controller.

The first one is to deal with the nonlinearity of dynamics of the manipulator. The second one is excitation of vibration in the transition period. The last difficulty is to cancel the disturbance during the manipulation and to control the force acting on the object. To deal with these difficulties, we proposed a hybrid controller composed of feedforward and feedback controllers. Feedforward commands are generated by inverse kinematics and inverse dynamics. The performances of the proposed controller are verified by numerical simulations.

REFERENCES

[1] Asada, H., Z.-D. Ma, and H. Tokumaru, "Inverse Dynamics of Flexible Robot Arms: Modeling and Computation for Trajectory Control," *ASME Journal of Dynamic Systems, Measurement, and Control*, 112, pp.177-185, 1990

[2] Bayo, E., P.Papadopoulos, J.Stubbe, and M.A.Serna, "Inverse Dynamics and Kinematics of Multi-link Elastic Robots: An Iterative Frequency Domain Approach," *International Journal of Robotics Research*,8(6), pp.49-62, 1989

[3] Dai, Y.Q.,K.Usui, and M.Uchiyama, "A New Iterative Approach Solving Inverse Flexible-Link Manipulator Kinematics," *Proceedings of the 35th IEEE Conference on Decision and Control*, pp.2493-2494, 1996

[4] R.E.Goddard, Y.F.Zheng and H.Hemami, "Dynamic Hybrid Velocity/Force Control of Robot Compliant Motion over Globally Unknown Objects," IEEE Trans. on Robotics and Automation, Vol.8, No.1, pp. 132-138, 1992

[5] T.Yoshikawa and A.Sudou. "Dynamic Hybrid Position/Force Control of Robot Manipulators –On-Line Estimation of Unknown Constraint," IEEE Trans. on Robotics and Automation, Vol.9, No.2, pp.220-227, 1993

[6] D.Wang and N.H.McClamroch, "Position and Force Control for Constrained Manipulator Motion: Lyapunov's Direct Method," IEEE Trans. on Robotics and Automation, Vol.9, No.3, pp.308-313, 1993

[7] D.Jeon and M.Tomizuka, "Learning Hybrid Force and Position Control of Robot Manipulators," IEEE Trans. on Robotics and Automation, Vol.9, No.4, pp.423-431, 1993

A Appendix

Body 2 is divided into N finite elements where are numbered as $1, 2, \cdots, N$ from joint 2 to the end effector. The nodes are also numbered as $0, 1, \cdots, N$ from joint 2 to the end effector. An elastic deformation in element $n, w^{(2)}_{2,n}$ is expressed as

Element 1

$$w^{(2)}_{2,1}(t,\rho^{(2)}_1) = \left[x_n - \frac{2}{l}x_n^2 + \frac{1}{l^2}x_n^3 ,\ \frac{3}{l^2}x_n^2 - \frac{2}{l^3}x_n^3 ,\ -\frac{1}{l}x_n^2 + \frac{1}{l^2}x_n^3 \right] \begin{bmatrix} w'^{(2)}_{2,0}(t) \\ w^{(2)}_{2,1}(t) \\ w'^{(2)}_{2,1}(t) \end{bmatrix} \quad (24)$$

where,

$$x_1 = \begin{cases} \rho^{(2)}_1 & ; 0 < \rho^{(2)}_1 < l \\ 0 & ; l < \rho^{(2)}_1 \end{cases}$$

$$l = \frac{r^{(2)}_1}{N-2}$$

Element $2 \sim N-2$

$$w^{(2)}_{2,n}(t,\rho^{(2)}_1) = \begin{bmatrix} x_n^0 - \frac{3}{l^2}x_n^2 + \frac{2}{l^3}x_n^3 \\ x_n - \frac{2}{l}x_n^2 + \frac{1}{l^2}x_n^3 \\ \frac{3}{l^2}x_n^2 - \frac{2}{l^3}x_n^3 \\ -\frac{1}{l}x_n^2 + \frac{1}{l^2}x_n^3 \end{bmatrix}^T \begin{bmatrix} w^{(2)}_{2,n-1}(t) \\ w'^{(2)}_{2,n-1}(t) \\ w^{(2)}_{2,n}(t) \\ w'^{(2)}_{2,n}(t) \end{bmatrix} \quad (25)$$

where,

$$x_n = \begin{cases} 0 & ; 0 < \rho_1^{(2)} < (n-1)l \\ \rho_1^{(2)} - (n-1)l & ; (n-1)l < \rho_1^{(2)} < nl \\ 0 & ; nl < \rho_1^{(2)} \end{cases}$$

Element $N-1$

$$w_{2,n}^{(2)}(t, \rho_1^{(2)}) = \left[x_{N-1}^0 \ , \ x_{N-1} \right] \begin{bmatrix} w_{2,N-2}^{(2)}(t) \\ w_{2,N-1}^{\prime(2)}(t) \end{bmatrix} \quad (26)$$

where,

$$x_{N-1} = \begin{cases} 0 & ; 0 < \rho_1^{(2)} < (N-2)l \\ \rho_1^{(2)} - (N-2)l & ; (N-2)l < \rho_1^{(2)} < R^{(3)} \\ 0 & ; R^{(3)} < \rho_1^{(2)} \end{cases}$$

$\widehat{w}_{2,n}^{(2)}$: elastic deformation of element n at node n
$\widehat{w}_{2,n}^{\prime(2)}$: angle of rotation of element n at node n
At nodes 0 and N, we may set the condition that

$$\widehat{w}_0^{(2)} = \widehat{w}_N^{(2)} = 0$$

Then, an elastic deformation in body 2 is expressed as

$$w_2^{(2)}(t, \rho_1^{(2)}) = B^{(2)}(\rho_1^{(2)}) \ \widehat{w}^{(2)}(t) \quad (27)$$

$$B^{(2)} = \begin{bmatrix} 0 \\ B_2^{(2)} \\ 0 \end{bmatrix}$$

$$B_2^{(2)} = \left[\widehat{b}_0' \ , \ \widehat{b}_1 \ , \ \widehat{b}_1' \ , \ \cdots \ , \ \widehat{b}_{N-2} \ , \ \widehat{b}_{N-2}' \ , \ \widehat{b}_{N-1}' \right]$$

$$\widehat{w}^{(2)T} = \left[\widehat{w}_{2,0}^{\prime(2)} \ , \ \widehat{w}_{2,1}^{(2)} \ , \ \widehat{w}_{2,1}^{\prime(2)} \ , \right.$$
$$\left. \cdots \ , \ \widehat{w}_{2,N-2}^{(2)} \ , \ \widehat{w}_{2,N-2}^{\prime(2)} \ , \ \widehat{w}_{2,N-1}^{\prime(2)} \right]$$