

SPACECRAFT 3-AXIS ATTITUDE CONTROL BY SPACE ROBOT MOTION

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ABSTRACT

This paper is concerned with a spacecraft attitude control by a space robot motion. The space robot motion is characterized by the nonholonomic nature, in which a closed trajectory in the robot joint space yields an attitude change of the spacecraft. Two dimensional, i.e., planar, robot motion has been extensively discussed by many authors. Especially, an approach using Green's Theorem has been widely applied since the attitude change of a spacecraft is given by line integrals along each joint trajectory. A robot motion with closed joint trajectories can be transferred to a surface integral. Based on the above considerations, a simple and approximate method for a spacecraft 3-axis attitude control is proposed in this paper. A space robot is mounted on a spacecraft and is assumed to have three rotary joints, yaw, pitch and roll. The spacecraft attitude will be controlled by the space robot motion where closed trajectory, cubic-like, in three dimensional joint space will be repeated cyclically. This corresponds to the closed trajectory motion on reduced three-two dimensional joint spaces and approximate method of two dimensional, planar, operation. A simulation result is also presented to show the validity of this approach.

I. INTRODUCTION

The nonholonomy is essential nature of free-flying space robotics and space articulated mechanisms. That is, depending on the trajectory of joint space, the attitude change of the body, which mounts robot and/or mechanism, will differ. This also shows the capability of attitude control by robot arm motion without using other devices, such as reaction wheels. Especially, free-flying space structure, if articulated mechanism like space

robot is mounted there, no other device is required to control the space structure.

In this paper an application of the nonholonomic nature to three-axis attitude control of the spacecraft is discussed utilizing motion of three degrees joint freedom, which is generally seen in usual space robot configuration. In nonholonomic case, attitude change of the spacecraft is given by line integrals along each joint trajectory. Cyclic closed joint operations can yield required attitude change. In two dimensional case it is easily shown that closed trajectory operation can be transferred to surface integral using Green's theorem. This paper applies the same idea as above to three dimensional case. Closed trajectory in three dimensional joint space will be reduced to three-two dimensional joint spaces by mapping. Obtaining closed trajectory on each two dimensional joint space, cubic-like closed trajectory will be generated and operated cyclically until desired attitude change is achieved. Also a numerical simulation result is given to validate our discussions. The approach shown in this paper is widely applicable to general free-flying space structure and mechanism.

II. ANALYTICAL DEVELOPMENT

Before proceeding to the general three dimensional case, we briefly discuss simple two dimensional case, i.e., planar motion of robot arm. Fig.1 illustrates a spacecraft with robot arm. There are one degree of freedom for the spacecraft attitude, θ , and two degrees of freedom for the robot joint, ϕ_1, ϕ_2 . As shown by many authors, e.g., reference [1], the attitude change,

$\Delta\theta$, due to the robot arm motion is represented by the line integrals along the arm joint trajectory.

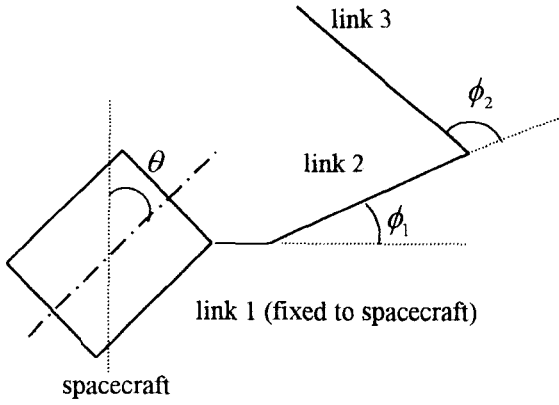


FIG.1 Definition of Planer Space Robot

$$\Delta\theta = \int_{\phi_{1i}}^{\phi_{1f}} A(\phi_1, \phi_2) d\phi_1 + \int_{\phi_{2i}}^{\phi_{2f}} B(\phi_1, \phi_2) d\phi_2 \quad (1)$$

where ϕ_{1i}, ϕ_{2i} are initial values, and ϕ_{1f}, ϕ_{2f} are final values of integration intervals. If the trajectory in the joint space is closed as seen in Fig.2, the attitude change of the spacecraft, along the trajectories I and II is given below.

$$\Delta\theta = \Delta\theta_1 - \Delta\theta_2 = \oint_{S(\Sigma)} [A(\phi_1, \phi_2) d\phi_1 + B(\phi_1, \phi_2) d\phi_2] \quad (2)$$

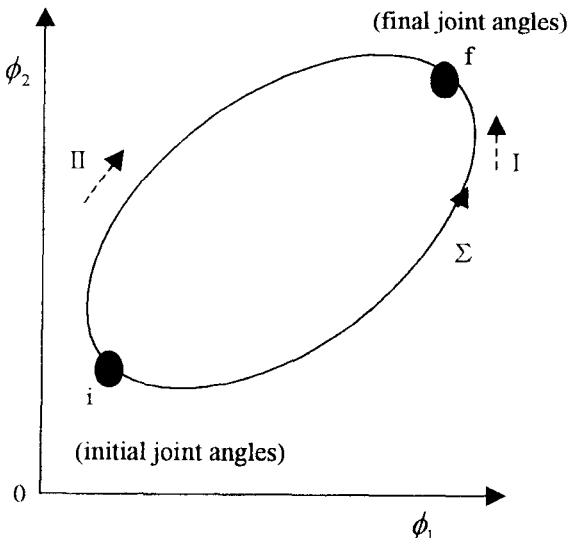


FIG.2 Closed Trajectory in Two Dimensional Joint Space

The above integral, can be transferred to the surface integral using Green's Theorem. That is,

$$\Delta\theta = \iint_{S(\Sigma)} \left[\left(\frac{\partial B(\phi_1, \phi_2)}{\partial \phi_1} \right) - \left(\frac{\partial A(\phi_1, \phi_2)}{\partial \phi_2} \right) \right] d\phi_1 d\phi_2 \quad (3)$$

where $S(\Sigma)$ equals to the area by the closed trajectory.

At this point if we define the configuration and mass properties of the robot arm, we will be able to obtain information on the integrand in eq.(3) over the entire operational region of the joint space. Fig.3 illustrates an example of contour map of the integrand.

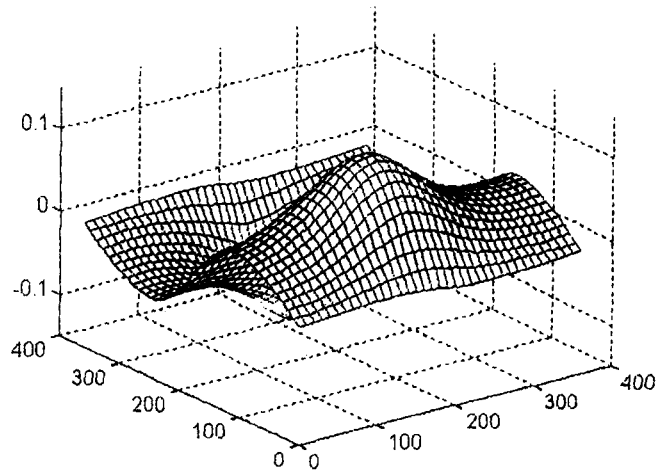


FIG.3 Illustrated Contour Map of Integrand

When we specify the closed trajectory in the joint space, the attitude change is available by calculating the eq.(3). A systematic way to generate attitude change of the spacecraft is to cyclically use an specific area on which the sign of the integrand is constant. This means a small closed trajectory robot arm motion will be repeated until enough attitude change is reached. The details of this feedback approach are given in reference [2].

Then, Let us consider a general space robot configuration, including a spacecraft, and joint allocation as shown in Fig.4. All the joints are assumed to be rotary without loss of generality. θ_x, θ_y and θ_z denote the spacecraft attitude angles around each axis. And ϕ_1 gives the shoulder yaw joint angle.

ϕ_2 and ϕ_3 represent the shoulder pitch and the elbow pitch joint angles.

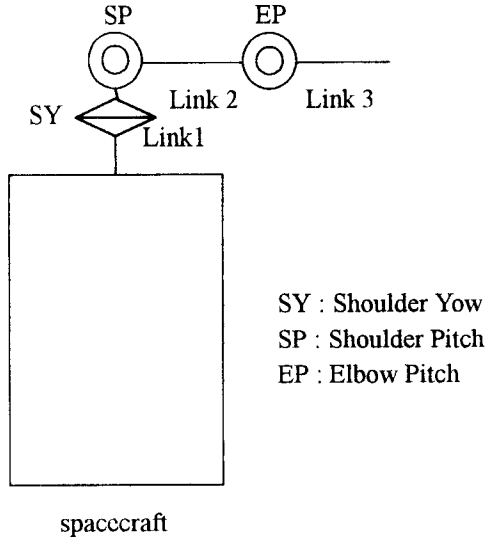


FIG.4 Definition of Three Dimensional Space Robot

As shown in previous two dimensional case, the spacecraft attitude changes, $\Delta\theta_x$, $\Delta\theta_y$ and $\Delta\theta_z$, are given by the following sum of line integrals:

$$\Delta\theta_x = \int_{\phi_{1i}}^{\phi_{1f}} A_x(\phi_1, \phi_2, \phi_3) d\phi_1 + \int_{\phi_{2i}}^{\phi_{2f}} B_x(\phi_1, \phi_2, \phi_3) d\phi_2 + \int_{\phi_{3i}}^{\phi_{3f}} C_x(\phi_1, \phi_2, \phi_3) d\phi_3 \quad (4)$$

$$\Delta\theta_y = \int_{\phi_{1i}}^{\phi_{1f}} A_y(\phi_1, \phi_2, \phi_3) d\phi_1 + \int_{\phi_{2i}}^{\phi_{2f}} B_y(\phi_1, \phi_2, \phi_3) d\phi_2 + \int_{\phi_{3i}}^{\phi_{3f}} C_y(\phi_1, \phi_2, \phi_3) d\phi_3 \quad (5)$$

$$\Delta\theta_z = \int_{\phi_{1i}}^{\phi_{1f}} A_z(\phi_1, \phi_2, \phi_3) d\phi_1 + \int_{\phi_{2i}}^{\phi_{2f}} B_z(\phi_1, \phi_2, \phi_3) d\phi_2 + \int_{\phi_{3i}}^{\phi_{3f}} C_z(\phi_1, \phi_2, \phi_3) d\phi_3 \quad (6)$$

And if we assume those are closed trajectories, then, applying Green's Theorem, we can rewrite above line integrals to the following sum of surface integrals.

$$\Delta\theta_i = \frac{1}{2} \oiint \left(\frac{\partial B_i}{\partial \phi_1} - \frac{\partial A_i}{\partial \phi_2} \right) d\phi_1 d\phi_2 + \frac{1}{2} \oiint \left(\frac{\partial C_i}{\partial \phi_2} - \frac{\partial B_i}{\partial \phi_3} \right) d\phi_2 d\phi_3 + \frac{1}{2} \oiint \left(\frac{\partial A_i}{\partial \phi_3} - \frac{\partial C_i}{\partial \phi_1} \right) d\phi_3 d\phi_1 \quad (7)$$

where $i = x, y, z$.

Let us assume small closed trajectory in each two dimensional joint space, mapped from three dimensional joint space. Then, we can take a linear approximation in the following vector and matrix form:

$$\begin{pmatrix} \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = \begin{bmatrix} M_{xAB} & M_{xBC} & M_{xCA} \\ M_{yAB} & M_{yBC} & M_{yCA} \\ M_{zAB} & M_{zBC} & M_{zCA} \end{bmatrix} \times \begin{pmatrix} \Delta\phi_1 \Delta\phi_2 \\ \Delta\phi_2 \Delta\phi_3 \\ \Delta\phi_3 \Delta\phi_1 \end{pmatrix} = [M] \begin{pmatrix} \Delta\phi_1 \Delta\phi_2 \\ \Delta\phi_2 \Delta\phi_3 \\ \Delta\phi_3 \Delta\phi_1 \end{pmatrix} \quad (8)$$

where abbreviations are $M_{LAB} = \frac{\partial B_i}{\partial \phi_1} - \frac{\partial A_i}{\partial \phi_2}$

($i = x, y, z$) and so forth. The left hand side of the

eq.(8) means small three-axis attitude change vector of the spacecraft, and The vector in the right hand side represents a mapped closed area from three dimensional to two dimensional joint space. As far as such small closed trajectory operations are concerned, linearity is almost preserved between those spacecraft attitude changes and closed area. This is a basic idea to derive a corrective approach of spacecraft attitude using small closed trajectory. And if we can specify nominal joint

angles around which each joint is operated, we will be able to obtain the information on elements of matrix M by calculating integrands in eq.(7).

$$\begin{pmatrix} \Delta\phi_1 & \Delta\phi_2 \\ \Delta\phi_2 & \Delta\phi_3 \\ \Delta\phi_3 & \Delta\phi_1 \end{pmatrix} = [M]^{-1} \begin{pmatrix} \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \quad (9)$$

Of course, we have to assure the invertibility of the matrix. This will depend on joints freedom allocation and arm configuration. And then, we can get information on three-two dimensional areas on each reduced, two dimensional joint space. Each joint travel is illustrated in Fig.5, where closed joint trajectory, a-b-c-d-e-f-a, will be projected onto each two dimensional joint space. In order to keep the linear approximation and achieve the convergence to the desired spacecraft attitude angle, we have to limit the area of each joint closed trajectory within small enough, therefore, a small constant gain k is introduced:

$$\begin{pmatrix} \Delta\phi_1 & \Delta\phi_2 \\ \Delta\phi_2 & \Delta\phi_3 \\ \Delta\phi_3 & \Delta\phi_1 \end{pmatrix} = k[M]^{-1} \begin{pmatrix} \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \quad (10)$$

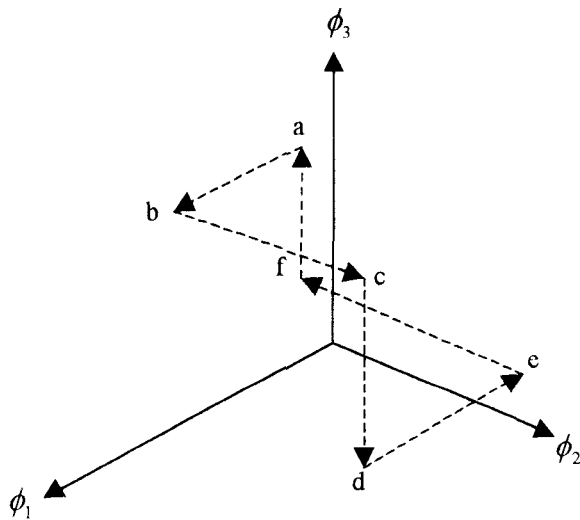


FIG.5 Closed Trajectory in 3 dimensional Joint Space

III. SIMULATION RESULT

In the preceding section, we have derived robot arm motion to achieve three axis attitude change of a spacecraft. A small closed trajectory, cubic-like, in the three dimensional joint space will be operated repeatedly to give small attitude change of the spacecraft. This is an approximate and extended method of the two dimensional ,planer, robot motion. In this

section we will show a numerical simulation result. Table 1 gives numerical data for physical properties of spacecraft and robot. Our simulation was to obtain 5 degree attitude change about each spacecraft body axis by space robot arm motion.

TABLE 1

	Spacecraft	Link 1	Link 2	Link 3
mass(kg)	2000.0	20.0	50.0	50.0
I (m)	3.5	0.25	2.5	2.5
I_x (kgm ²)	1400.0	0.1	0.25	0.25
I_y (kgm ²)	1400.0	0.1	26.0	26.0
I_z (kgm ²)	2040.0	10.0	26.0	26.0

Figs.6-8 illustrate calculated integrands in eq.(7) around specific joint angles. These values on the nominal joint angles represented those elements of the matrix in the right hand side of eq.(8) and its inversion in eq.(10). A small gain k in eq.(10) is assigned to be 0.15. In every cycle required attitude change vector, after each attitude correction, was given and, using eq.(10), three dimensional closed trajectory was generated and operated as shown in Fig.9. A history of attitude change is illustrated in Fig.10. As easily seen, a smooth attitude history has been obtained. This gives one aspect of the validity of our approach.

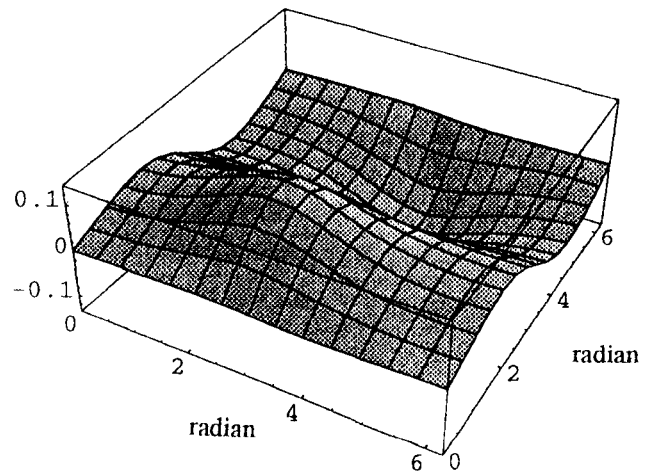
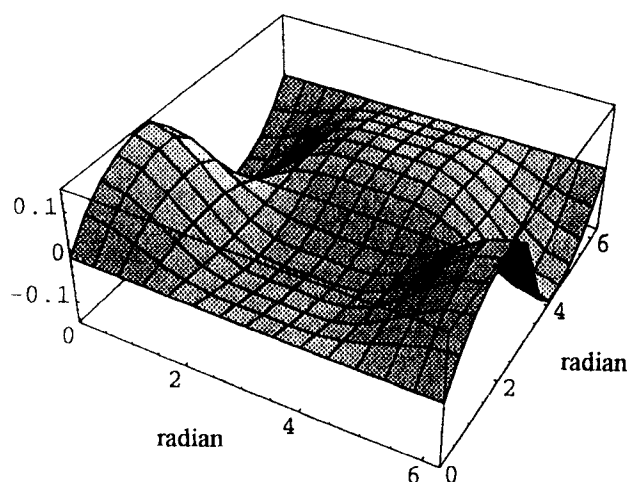
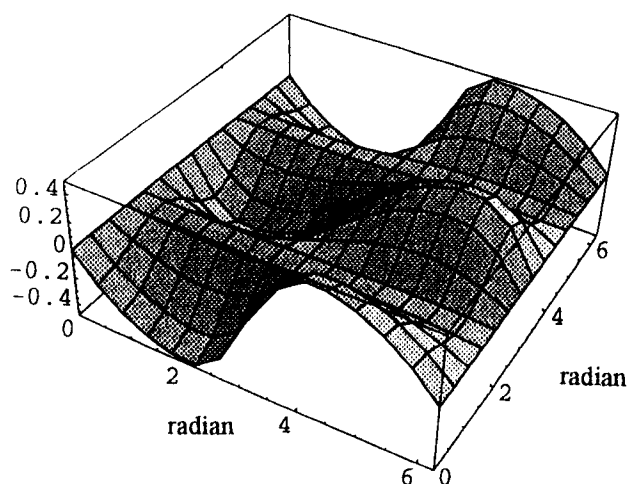


FIG.6 Contour Map for ϕ_2 and ϕ_3

FIG.7 Contour Map for ϕ_1 and ϕ_3 FIG.8 Contour Map for ϕ_1 and ϕ_2

IV. CONCLUSION

This paper is concerned with three axis attitude control of a spacecraft by a space robot motion. Three joint freedom is assumed to operate the space robot. Basically, an extension of two dimensional, planer, space robot operation is applied to three dimensional space robot motion strategy. Small closed trajectories in the three dimensional joint space is repeated to obtain small spacecraft attitude change. Cyclic motions of space robot eventually achieve required attitude change of the spacecraft.

Analytically, the above means three dimensional closed trajectory motion is projected onto two dimensional

joint space. And the approach derived from Green's Theorem is applied to transfer line integrals to surface integrals. Integrands of surface integrals are calculated around nominal joint angles and used to obtain an approximate closed area of each joint trajectory.

A numerical simulation was conducted to demonstrate the validity of our approach. Cubic trajectories in three dimensional joint space were generated and required attitude change of the spacecraft was achieved without unstable behavior.

Although the control method in this paper is based upon the linear approximation, three axis attitude of a spacecraft is well controlled by three joint freedom. This approach will be also applicable to the control of large space structure with articulated mechanism, so that no other device like momentum wheel will not be required.

REFERENCES

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- [2]S. Tsuda et. al., "Nonholonomic Space Robot Path Planning for Spacecraft Attitude Stabilization and Control", The Institute of Space and Astronautical Science Report No. 670, September 1997

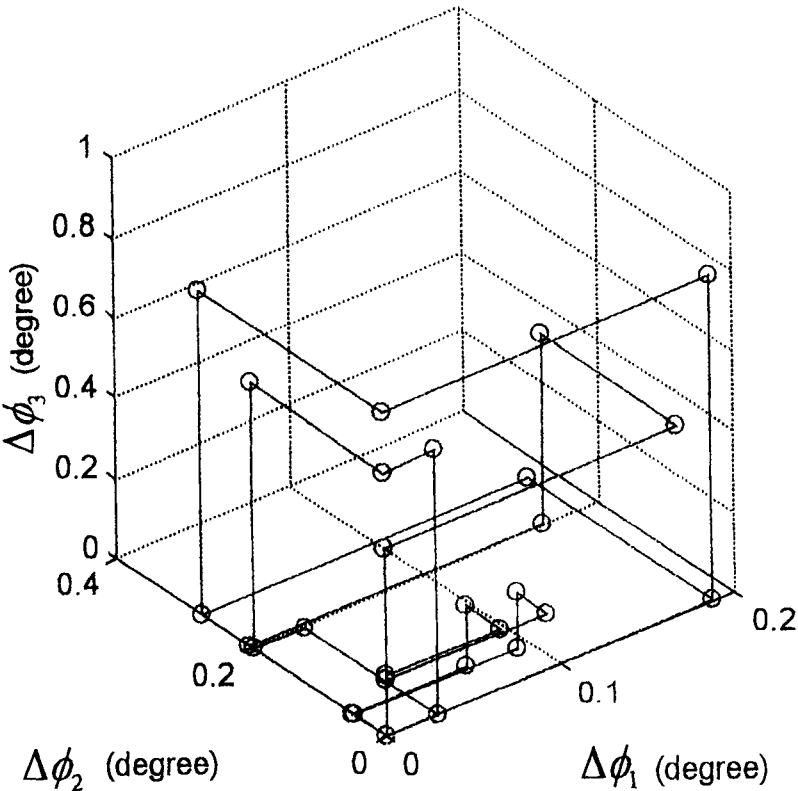


FIG.9 Closed Trajectory in Three Dimensional Joint Space for Numerical Simulation Case
($\Delta\phi_1$, $\Delta\phi_2$, $\Delta\phi_3$ is given around nominal joint angles)

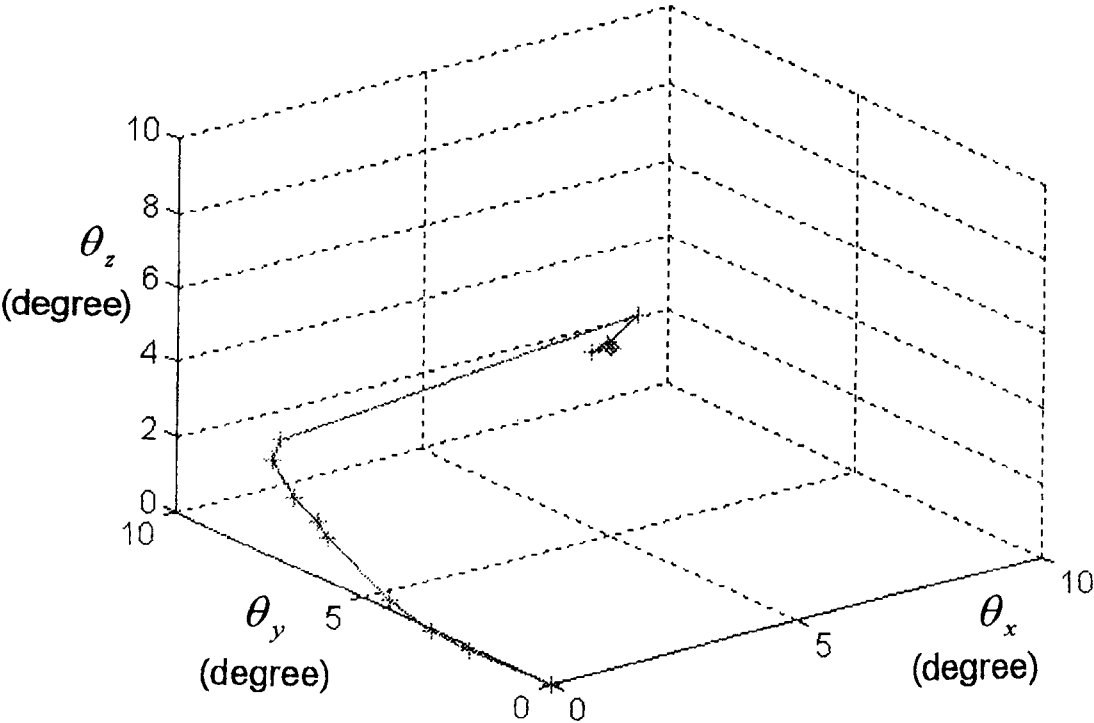


FIG.10 Spacecraft Attitude Change Profile by Repeated Closed Trajectory Arm Motion