Reactionless Manipulations and Proposal to ETS-VII On-Board Experiments

Kazuya Yoshida
Department of Aeronautics and Space Engineering, Tohoku University
Aoba 01, Sendai 980-8579, Japan
yoshida@astro.mech.tohoku.ac.jp

Dragomir N. Nenchev
Department of Intelligent Machines and System Engineering, Hirosaki University
Bunkyo-cho 3, Hirosaki 036-8561, Japan
nenchev@cc.hirosaki-u.ac.jp

Abstract

Space robots provides an interesting characteristics as under-actuated systems. If we operate a space manipulator without a good understanding on the under-actuated dynamics, the operatinal performance would be much degraded. However with a good understanding on the dynamics, we could operate the manipulator not disturbing the attitude or exciting the vibrations on its foot base. In this paper we propose such nice operation named as Reactionless Manipulations to be examined on the ETS-VII, a free-flying space robot developed by NASDA, currently flying in orbit.

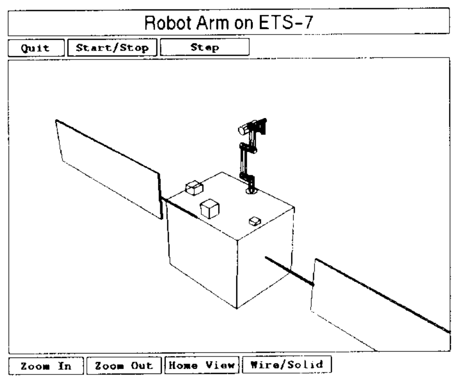

Figure 1 An example of Free-Floating Space Manipulator

1 Introduction

The interest toward complex robot systems is expanding for new application areas including space robots. A class of such robot systems are so-called under-actuated systems, characterized by the number of control actuators being less than the number of degree of freedom.

One typical example is a free-flying space manipulator (see Figure 1), in which the number of controllable joints are n in general but the number of system DOF is $n + 6$ including the position and orientation of the base body in the inertial space.

Another example of the under-actuated system is a dextrous manipulator arm mounted on a passive flexible base (see Figure 2). In literature, such a system is known under the name of long-reach manipulator [1], or flexible structure mounted manipulator system (FSMS) [2], and in this paper we simply say flexible-base manipulator.

This class of manipulator systems are regarded as a version of macro-mini manipulators. In space applications particularly, the dextrous manipualtor system for the international space station, SPDM [3], mounted on the endtip of the space station remote manipulator system, SSRMS, is a good example. A manipulator system on the Japanese module, JEM-

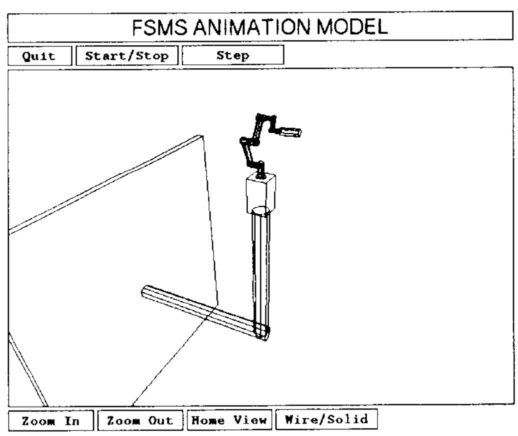

Figure 2 An example of Flexible-Base Manipulator

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RMS, also takes the macro-mini configuration. When all of active joints work simultaneously and coordinately, these systems are not an under-actuated system. But in the situation of fine positioning after coarse maneuver for example, the joints in the macro part are break locked and only the mini part will be operated most likely. The system is consided as an under-actuated system in such a situation, because the macro part behaves as a passive flexible structure.

One of major issues on the flexible-base manipulators is dynamic interaction between the motion of the dexterous manipulator, the mini part, and the vibrations of the flexible base, the macro part, due to the induced reaction. Such vibrations degrade the operational accuracy and settling time of the end point. One can find that the problem here is similar to the problem in free-floating manipulators at the point that the dynamic reaction of the manipulator arm induces the interactive motion in the supporting base, which is usually undesirable to dexterous operations. The difference between these two systems is that the manipulator base of a free-flying manipulator is a floating inertia, but that of a flexible-base manipulator is an inertia-spring-damper system.

The present authors formulated the dynamic equations for these under-actuated systems, paying attention to the commonality and difference between the free-flying manipulators and the flexible-base manipulators [4][5]. For the commonality, both systems are the first-order non-holonomic system, in which the coupling dynamics is expressed by the momentum equation as the first-order integral.

Getting an insight into this coupling dynamics, the authors discovered that there is a class of manipulator operation to de-couple the manipulator and base dynamics. A mathematical formulation to generate the decoupled dynamics is termed "Reaction Null-Space" (RNS) and such manipulation as "Reactionless Operation," or such motion trace as "Reactionless Path" [6]–[9].

If we operate the manipulator arm mounted on a floating or flexible base along a reactionless path, the manipulator generates zero reaction to the base, then no vibrations on the base and no degradation in the manipulation performance are expected.

The reactionless operation has been experimentally verified with a series of laboratory test bed of a flexible-base manipulator, named TREP-I and II [10] [12]. However the operation has not been tested on a free-flying manipulator because a free-flying experiment is very difficult in one-G environment. But now, we have a real free-flying space robot in orbit, the robot is the Engineering Test Satellite (ETS-VII) developed and operated by the National Space Development Agency (NASDA), Japan.

This paper aims to propose the reactionless operation as one of meaningful flight experiments of ETS-VII, in terms for better understanding of the space robot dynamics and better performance of orbital manipulation, with less attitude disturbance and less fuel/time consumption for attitude maintenance.

2 Formulation of Dynamics

2.1 Equation of Motion in General Form

Let us begin with a general discussion considering a system which motion is described by n degrees of freedom of the generalized coordinate $\boldsymbol{q} \in R^n$ for *active* joints and m degrees of freedom of the generalized coordinate $\boldsymbol{p} \in R^m$ for *passive* joints. Now, define $\boldsymbol{F}_q$ as active force/torque (twist) generated on coordinate $\boldsymbol{q}$, and $\boldsymbol{F}_p$ as a passive force/torque exerted on coordinate $\boldsymbol{p}$. Also, define $\boldsymbol{x}$ as a coordinate of a point of interest (the operational coordinate) composed by $\boldsymbol{p}$ and $\boldsymbol{q}$, and let an external force/torque $\boldsymbol{F}_{ex}$ be applied on $\boldsymbol{x}$. Hence, the applied external force/torque is decomposed as $\boldsymbol{J}_q^T \boldsymbol{F}_{ex}$ and $\boldsymbol{J}_p^T \boldsymbol{F}_{ex}$ onto each gereralized coordinate using corresponding Jacobian matrices.

The equation of motion of such system is generally expressed as:

$$\begin{bmatrix} \boldsymbol{H}_p & \boldsymbol{H}_{pq} \\ \boldsymbol{H}_{pq}^T & \boldsymbol{H}_q \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{p}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_p \\ \boldsymbol{c}_q \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_p \\ \boldsymbol{F}_q \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}_p^T \\ \boldsymbol{J}_q^T \end{bmatrix} \boldsymbol{F}_{ex} \tag{1}$$

where $\boldsymbol{H}_p$, $\boldsymbol{H}_q$, $\boldsymbol{H}_{pq}$ are inertia matrices. $\boldsymbol{c}_p$, $\boldsymbol{c}_q$ are non-linear Coriolis and centrifugel forces and they can include gravity forces if necessary.

Kinematic relationship among $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{x}$ is expressed using Jacobians as:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_p \dot{\boldsymbol{p}} + \boldsymbol{J}_q \dot{\boldsymbol{q}} \tag{2}$$

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}_p \ddot{\boldsymbol{p}} + \dot{\boldsymbol{J}}_p \dot{\boldsymbol{p}} + \boldsymbol{J}_q \ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_q \dot{\boldsymbol{q}} \tag{3}$$

The above set of equations are commonly applicable for any type of under-actuated manipulator systems.

2.2 For Free-Floating Manipulators

Now, let us consider a free-floating system composed by a single robot base which is floating in the inertial space without any external force or torque, and a serial manipulator arm at which end point any external force/torque is not apply. For such a space manipulator, the equation of motion is obtained from Equation (1) by replacing the symbols as Table 1.

We then obtain the following equations:

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}}_b \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_b \\ \boldsymbol{c}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\tau} \end{bmatrix} \tag{4}$$

Table 1 Symbol replacement for a free-flying manipulator

$\boldsymbol{p}$	$\rightarrow$	$\boldsymbol{x}_b$	: the position/orientation of the floating base
$\boldsymbol{q}$	$\rightarrow$	$\boldsymbol{\phi}$	: the joint angle of the arm
$\boldsymbol{x}$	$\rightarrow$	$\boldsymbol{x}_h$	: the position/orientation of the manipulator end point
$\boldsymbol{F}_p$	$\rightarrow$	$\boldsymbol{0}$	: the external force/torque on the base
$\boldsymbol{F}_q$	$\rightarrow$	$\boldsymbol{\tau}$	: the joint torque of the arm
$\boldsymbol{F}_{ex}$	$\rightarrow$	$\boldsymbol{0}$	: the external force/torque on the end point

$$\dot{\boldsymbol{x}}_h = \boldsymbol{J}_m\dot{\boldsymbol{\phi}} + \boldsymbol{J}_b\dot{\boldsymbol{x}}_b \tag{5}$$

$$\ddot{\boldsymbol{x}}_h = \boldsymbol{J}_m\ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{J}}_m\dot{\boldsymbol{\phi}} + \boldsymbol{J}_b\ddot{\boldsymbol{x}}_b + \dot{\boldsymbol{J}}_b\dot{\boldsymbol{x}}_b \tag{6}$$

For a space free-floating manipulator, there is no gravity exerting on the system, then the non-linear term becomes $\boldsymbol{c}_b = \dot{\boldsymbol{H}}_b\dot{\boldsymbol{x}}_b + \dot{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\phi}}_b$. Integrating the upper set of equation in (4) with respect to time, we obtain the total momenta of the system as:

$$\mathcal{L} = \boldsymbol{H}_b\dot{\boldsymbol{x}}_b + \boldsymbol{H}_{bm}\dot{\boldsymbol{\phi}} = \text{const.} \tag{7}$$

This equation attributes an important characteristics to the free-flying system.

2.3 Reaction Null-Space

The "Reaction Null-Space" is a useful idea to discuss the coupling and decoupling of dynamic interaction between a manipulator and its base. The reaction null-space concept has its roots in the earlier work on free-flying space manipulator by Nenchev et al, where the Fixed-Attitude-Restricted (FAR) Jacobian has been proposed as means to plan and control manipulator motion that does not disturb the attitude of the free-floating base [6].

In Equation (7), if the base does not have any motion, say $\dot{\boldsymbol{x}}_b = \boldsymbol{0}$, then the momenta comes from only the manipulator motion. The partial momenta for the manipulator part becomes:

$$\mathcal{L}_m = \boldsymbol{H}_{bm}\dot{\boldsymbol{\phi}} = \text{const.} \tag{8}$$

It is seen that a constant $\mathcal{L}_m$ indicates no motion of the base $\dot{\boldsymbol{x}}_b = \boldsymbol{0}$, then no reaction force or torque is present to yield the base motion.

In case the number of degrees of freedom of the active manipulator joints n is grater than that of the base coordinate m, the solution for the manipulator operation to satisfy $\mathcal{L}_m$ =const is given by:

$$\dot{\boldsymbol{\phi}}_c = \boldsymbol{H}_{bm}^+\mathcal{L}_m + (\boldsymbol{E} - \boldsymbol{H}_{bm}^+\boldsymbol{H}_{bm})\boldsymbol{\xi} \tag{9}$$

where $(\cdot)^+$ indicates pseudo-inverse, $\boldsymbol{\xi} \in R^n$ is an arbitrary vector.

The component $(\boldsymbol{E}-\boldsymbol{H}_{bm}^+\boldsymbol{H}_{bm})$ suggests the mapping onto the null space of the inertia matrix $\boldsymbol{H}_{bm}$ and this inertial null space is termed "Reaction Null-Space."

In the special case when $\mathcal{L}_m = \boldsymbol{0}$, Equation (9) becomes much simpler as:

$$\dot{\boldsymbol{\phi}}_{ns} = (\boldsymbol{E} - \boldsymbol{H}_{bm}^+\boldsymbol{H}_{bm})\boldsymbol{\xi}. \tag{10}$$

As long as we operate the manipulator using the joint velocities given by (10), no reaction force or torque is generated on the base, therefore no reactive motion or vibration is oscillated in the base. If integrable, the integration of (10) yields "Reactionless Paths," the trace of the manipulator motion which does not excite the base motion.

On the other hand, the first term of Equation (9) suggests maximum interaction with the base, in contrast with the second term for the reaction null-space. This maximum interaction characteristics can be used to an effective damping of the base vibration for flexible-base systems. For example, using the measurement of the base displacement $\Delta\boldsymbol{x}_b$ as a feedback signal and G as a gain matrix, we have a simple, but effective vibration surpression law:

$$\dot{\boldsymbol{\phi}}_v = G\boldsymbol{H}_{bm}^+\Delta\boldsymbol{x}_b \tag{11}$$

The above control space is perpendicular to the reaction null-space. Therefore these two operations (10) and (11) can be easily superimposed without interfering each others, just by simple addition:

$$\dot{\boldsymbol{\phi}}_c = \dot{\boldsymbol{\phi}}_v + \dot{\boldsymbol{\phi}}_{ns}$$

$$\dot{\boldsymbol{\phi}}_c = g\boldsymbol{H}_{bm}^+\Delta\boldsymbol{x}_b + (\boldsymbol{E} - \boldsymbol{H}_{bm}^+\boldsymbol{H}_{bm})\boldsymbol{\xi} \tag{12}$$

For a space robot which has a 6 DOF manipulator arm on a floating base satellite, $n = 6$ and $m = 6$, therefore the reaction null-space does not exist in general. However, if we care the base attitude only, allowing the base translation during the manipulation, then $m = 3$ and we have the reaction null-space and can find reactionless paths of the manipulator arm.

3 Flight Experiment Opportunity on ETS-VII

The Engineering Test Satellite VII (ETS-VII), developed by National Space Development Agency of

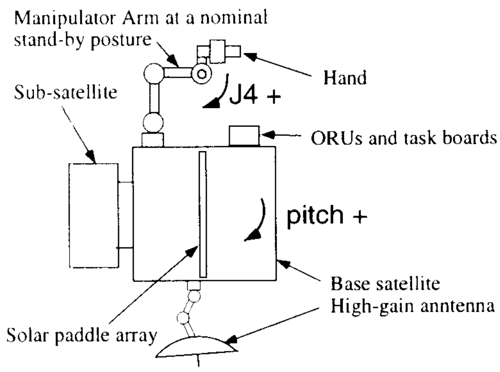

Figure 3 A schematic configuration of ETS-VII

Japan (NASDA) and launched on November 1997, is currently flying on orbit and conducting a lot of interesting experiments with a 2 meter-long, 6 DOF manipulator arm mounted on its un-manned spacecraft. The ETS-VII should be noted as one of remarkable outcomes of research effort on space robots, characterized as a free-flying manipulator system. A plenty of fresh results from ETS-VII will be reported in this symposium.

The officially prepared experiments are completed by the end of May 1999, while the satellite lives in a health condition. Making most of this unique opportunity that a mankind has an operational robot in orbit, NASDA officials recently announced to accept proposals for meaningful options of experiments. And the present authors have proposed the reactionless manipulation, and related operations, to be test by ETS-VII in orbit.

4 Proposed Experiments

The proposed experiments consist of two types of maneuvers for the on-board manipulator. One is the reactionless manipulation, and the other is the attitude change maneuver by the manipulator reaction.

4.1 Reactionless Manipulations

As is presented in Section 2, the reactionless manipulations are obtained by Equation (10) or

$$\mathcal{L}_m = \boldsymbol{H}_{bm}\dot{\boldsymbol{\phi}} = \boldsymbol{0}, \tag{13}$$

provided a non-empty null-space of $\boldsymbol{H}_{bm}$ and an arbitrary $\boldsymbol{\xi}$.

In practice, we care 3 DOF of attitude disturbance of the base for 6 DOF of the on-board manipulator, then 3 residual DOF are left in $\boldsymbol{\xi}$ and the reactionless manipulations exist in general. In order to uniquely determine $\boldsymbol{\xi}$, a relationship with 3 degrees of net freedom can be accepted. Kinematic relationships of the manipulator are such candidate, if paying attention to 3 degrees positions or orientations:

$$\boldsymbol{v}_h = \boldsymbol{J}_{mv}\dot{\boldsymbol{\phi}} \tag{14}$$

or

$$\boldsymbol{\omega}_h = \boldsymbol{J}_{m\omega}\dot{\boldsymbol{\phi}} \tag{15}$$

where $\boldsymbol{J}_{mv}$ and $\boldsymbol{J}_{m\omega}$ are partial manipulator Jacobians for the linear and angular velocity of the manipulator end tip, respectively.

Specification of $\boldsymbol{\xi}$ with Equation (14) or (15) is alternatively computed from a direct combination of (13) and (14) or (15), yielding the following solutions:

$$\dot{\boldsymbol{\phi}} = \begin{bmatrix} \tilde{\boldsymbol{H}}_{bm} \\ \boldsymbol{J}_{mv} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{v}_h \end{bmatrix} \tag{16}$$

or

$$\dot{\boldsymbol{\phi}} = \begin{bmatrix} \tilde{\boldsymbol{H}}_{bm} \\ \boldsymbol{J}_{m\omega} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\omega}_h \end{bmatrix} \tag{17}$$

where $\tilde{\boldsymbol{H}}_{bm} \in R^{3\times n}$ is modified from $\boldsymbol{H}_{bm} \in R^{n\times n}$ in order to focus the attitude motion of the base. Here $n = 6$ then the matrices

$$\begin{bmatrix} \tilde{\boldsymbol{H}}_{bm} \\ \boldsymbol{J}_{mv} \end{bmatrix} \text{ and } \begin{bmatrix} \tilde{\boldsymbol{H}}_{bm} \\ \boldsymbol{J}_{m\omega} \end{bmatrix}$$

are square in 6×6 and conventional inverse exists if non-singular.

Computation of the reactionless manipulations by Equations (16) and (17) are much more simple and practical than by Equation (10) with pseudo inverse and unspecified $\boldsymbol{\xi}$. For further smooth computation of the conventional inverse continuously along a sequence of motion, the method with an adjoint matrix [13] which is originally proposed for on-line teleoperation can be applied. The method gives a solution for $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$ with a square matrix $\boldsymbol{A}$ by

$$\boldsymbol{x} = k \cdot \mathrm{adj}(\boldsymbol{A})\boldsymbol{y} \tag{18}$$

where k is an arbitrary constant. This guarantees smooth solutions on and around the singularity with a proper choice of k. And $k = 1/\det(\boldsymbol{A})$ provides the conventional inverse.

4.2 Specific Proposal and Expected Results

A specific reactionless path of the ETS-VII manipulator arm is computed with Equation (17) from the nominal "stand-by" position toward a positive pitch angle of the hand, while yaw and roll of the hand are kept zero and the positions of the hand are left arbitrary. See Figure 3. Such a motion trajectory in the joint space is depicted in Figure 4, named "Path

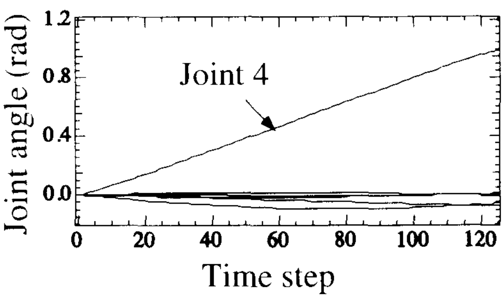

Figure 4 A reactionless maneuver, Path A

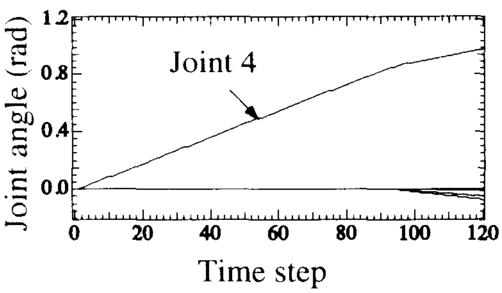

Figure 5 A trivial maneuver, Path B

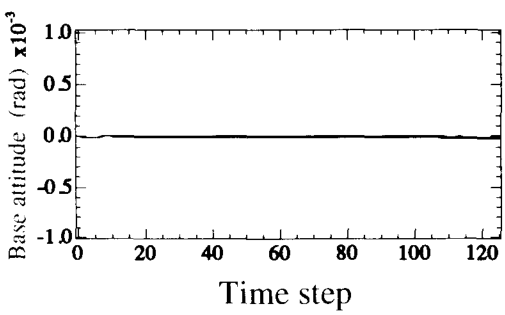

Figure 6 The reaction on the base attitude in Path A operation

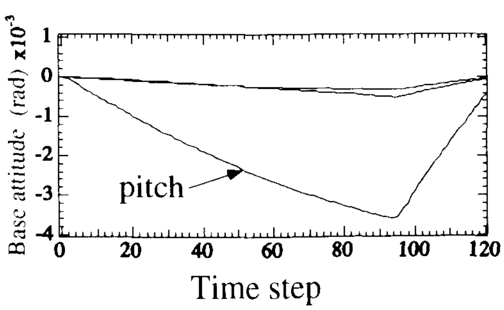

Figure 7 The reaction on the base attitude in Path B operation

A," where the joint 4 rotates by 50 degrees playing a major role, but other joints also move coordinately to counterbalance the reaction.

For comparison, a trivial operation is planned as shown in Figure 5, named "Path B," where only joint 4 rotates by 50 degrees first, then an adjustive motion by all joints is made to get the same destination as Path A.

The expected reaction of the base satellite in the absence of the attitude control system are simulated and depicted in Figure 6 and 7 for each manipulation. The simulation is carried out with the software package developed by the Space Robotics Lab. of Tohoku University, "The SpaceDyn," a MATLAB tool box for space and mobile robots [14] with a relatively precise parameters of ETS-VII.

In these figures, the difference is remarkable with exactly zero reaction in Path A, against non-negligible attitude degradation (0.2 degrees in pitch) in Path B. The proposed motion looks very similar to a frequently observed operation in the manipulation of ETS-VII, when the arm is operated from the nominal stand-by to a posture ready to pick-up an on-board component. Our proposing experiment would prove that a reactionless maneuver, obtained by a small modification from a trivial trajectory, could result in a remarkable stability of the base and saving of the time and energy to recover from the attitude disturbance.

4.3 Attitude Maneuver by Manipulator Reaction

The attitude change or control maneuvers by an effective usage of the manipulator reaction is also proposed. On the contrary to the reactionless maneuvers, a maximum coupling between the manipulator reaction and the base attitude is effective for attitude maneuvers. Such operations are obtained from Equation (11). A good example will be proposed for a possible flight experiment on ETS-VII.

5 Conclusion

In this paper, we propose the reactionless manipulations for a free-flying space robot that would not disturbe the base attitude. In the former half of the paper, we present theoretical backgound and derivation of the reactionless manipulations and related concepts. In the latter half, we propose a specific motion trajectory to yiled reactionless maneuver for a possible flight experiment on ETS-VII, a free-flying space robot currently in operation. Dynamic simulations are carried out to check the expected results in the flight experiments. It is clearly seen that the reactionless maneuver in which all joints move coordinately to counterbalance the reaction would yield completely non-zero disturbance on the base, while a trivial maneuver in most of which only one joint moves at a time, yields non-negligible attitude disturbance.

We do hope the proposed experiments are carried out in orbit by ETS-VII soon.

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