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Reduction of Control Torque for Space Manipulators Using Dynamic Manipulability Ellipsoid

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Abstract

This paper proposes a method of on-line motion control of a free-flying space robot to reduce the joint torque required for hand manipulation. The proposed method is based only on the local state of joint angles and utilizes redundant degrees of freedom in the manipulator motion to generate the desired trajectories of joint angles that control the major axis of the dynamic manipulability ellipsoid (DME) in the direction of the end-effector's motion as close as possible. The present method is applied to the 2D model of a 3-link space manipulator and the results are compared with the solution of the optimal control. The results of numerical simulation show that the proposed on-line control method is effective in reducing the control torque and that it decreases the angular error between the DME major axis and the direction of the end-effector's motion to align with each other.

1. Introduction

For future activities in space, the role of robotic manipulators carried by a spacecraft will be of importance. Such applications of free flying space manipulators are potentially very useful for performing complex tasks in space [1]. However, the practical applications will encounter many dynamic and control problems due to the dynamic coupling between the manipulator arm and the spacecraft main body [2]. Movement of a manipulator will disturb the attitude of the spacecraft and such coupling will reduce the life of the system by consuming excessive control fuel. Therefore, there exist the needs for designing the trajectory of the space manipulator that minimizes the control torque required for the end-effector's motion [3]. There are two different types of approaches to the energy efficient control of space manipulators. One is to solve the optimal control problem for given tasks, i.e., off-line path planning [4]. The other is to design the desired joint angular velocity using local information, i.e., on-line approach [5]. This paper presents an on-line method for determining joint angular velocities that performs the biobjectives of moving the hand to the final position and reducing the joint torque required for the manipulation.

2. Dynamic Manipulability Ellipsoid (DME)

Dynamic equation of motion of a space manipulator is given as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau} \tag{1}$$

where

$\mathbf{M}$: inertial matrix
$\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})$: centrifugal and Coriolis force
$\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}$: vector of joint angles, angular velocities and angular accelerations
$\boldsymbol{\tau}$: joint torque.

The relation between joint angular velocity $\dot{\mathbf{q}}$ and hand velocity $\mathbf{v}$ is given by the following equation:

$$\mathbf{v} = \mathbf{J}\dot{\mathbf{q}} \tag{2}$$

where $\mathbf{J}$ is generalized Jacobian matrix [1].

2.1 Definition of DME

Differentiating Eq. 2 and substituting $\ddot{\mathbf{q}}$ by Eq. 1 one obtains the following equation:

$$\dot{\tilde{\mathbf{v}}} = \mathbf{J}\mathbf{M}^{-1}\tilde{\tau} \tag{3}$$

where $\dot{\tilde{\mathbf{v}}}$ is (modified) hand acceleration and $\tilde{\boldsymbol{\tau}}$ is (modified) joint torque:

$$\dot{\tilde{\mathbf{v}}} = \dot{\mathbf{v}} - (\mathbf{I} - \mathbf{J}^{+}\mathbf{J})\dot{\mathbf{J}}\dot{\mathbf{q}} \tag{4}$$

$$\tilde{\boldsymbol{\tau}} = \boldsymbol{\tau} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{M}\mathbf{J}^{+}\dot{\mathbf{J}}\dot{\mathbf{q}} \tag{5}$$

and $\mathbf{J}^{+}$ denotes pseudo inverse of $\mathbf{J}$.

We here consider all set of hand acceleration that can be reached by using joint torque for $\|\tilde{\boldsymbol{\tau}}\| \le 1$. Then, we have the following relations:

$$\dot{\tilde{\mathbf{v}}}^{T}\mathbf{G}\dot{\tilde{\mathbf{v}}} \le 1 \tag{6}$$

$$\mathbf{G} = (\mathbf{J}^{+})^{T}\mathbf{M}^{T}\mathbf{M}\mathbf{J}^{+} \tag{7}$$

Thus the reachable set of hand acceleration forms an ellipsoid in the inertial space that is called as dynamic manipulability ellipsoid (DME). The length and direction of the principal axes are given as eigenvalues and eigenvectors of the matrix $\mathbf{G}$. If we consider planner motion of the manipulator, $\mathbf{G}$ is a 2×2 matrix.

The surface of DME represents the magnitude of hand acceleration that can be produced by using unit joint torque. That is, with given amount of input torque, the maximum acceleration can be generated in the direction of the major axis of DME and the possible acceleration is minimum in the direction of the minor axis. Therefore, if the shape of the DME is controllable by using redundant degrees of freedom in the manipulator motion, it is reasonable to control the DME major axis to direct to the target point of the hand motion. Thereby we can always obtain the maximum hand acceleration by using the given amount of joint torque.

2.2 Sensitivity of DME

If $m \times m$ symmetric matrix $\mathbf{A}$ is a function of ρ, then one derives partial derivative of the eigenvector of $\mathbf{A}$ with respect to ρ in the form as:

$$\frac{\partial \boldsymbol{\phi}_i^*}{\partial \rho} = \sum_{j=1}^{m} a_{ji}^* \boldsymbol{\phi}_j^* \tag{8}$$

$$a_{ji}^* = \frac{1}{\lambda_i^* - \lambda_j^*} \boldsymbol{\phi}_j^* \frac{\partial \mathbf{A}}{\partial \rho} \boldsymbol{\phi}_i^* \quad (i \ne j) \tag{9}$$

$$a_{ji}^* = 0 \quad (i = j) \tag{10}$$

where λ_i^*, $\boldsymbol{\phi}_i^*$ (i=1,2,…,m) are eigenvalues and eigenvectors of $\mathbf{A}$.

Since matrix $\mathbf{G}$ is a function of the spacecraft attitude angle q_0 and joint angles $\mathbf{q}$, one obtains the derivative of the eigenvector of $\mathbf{G}$ as:

$$\frac{\partial \boldsymbol{\phi}_i}{\partial \tilde{q}_k} = \boldsymbol{\Phi}\mathbf{a}^k \tag{11}$$

$$\boldsymbol{\Phi} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \cdots \boldsymbol{\phi}_m] \tag{12}$$

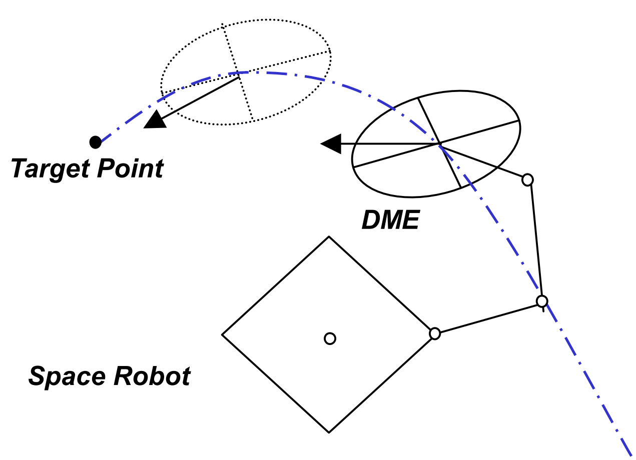

Fig. 1 Space robot and DME

$$\mathbf{a}^k = \lfloor a_{ji}^k \rfloor_j \tag{13}$$

$$a_{ji}^k = \frac{1}{\lambda_i - \lambda_j}\left[\boldsymbol{\phi}_j^T \frac{\partial \mathbf{G}}{\partial \tilde{q}_k} \boldsymbol{\phi}_i\right] \tag{14}$$

where $\tilde{\mathbf{q}} = [q_0, \mathbf{q}^T]^T$.

Defining the eigenvector that corresponds to the major axis of DME as $\boldsymbol{\phi}_i$, the time derivative of this eigenvector is obtained in the following form.

$$\begin{aligned}\frac{\partial \boldsymbol{\phi}_i}{\partial t} &= \sum_{k=0}^{n} \frac{\partial \boldsymbol{\phi}_i}{\partial \tilde{q}_k} \dot{\tilde{q}}_k \\ &= \sum_{k=0}^{n} \Phi \mathbf{a}^k \dot{\tilde{q}}_k \\ &= \Phi[[\mathbf{a}^1, \mathbf{a}^2, ..., \mathbf{a}^n]\dot{\mathbf{q}} + \mathbf{a}^0 \dot{q}_0]\end{aligned} \tag{15}$$

Conservation of angular momentum gives the relationship between spacecraft angular velocity $\dot{q}_0$ and joint angular velocity vector $\dot{\mathbf{q}}$ as [3]:

$$\dot{q}_0 = \mathbf{F}\dot{\mathbf{q}} \tag{16}$$

$$\mathbf{F} = [f_1, f_2, .., f_n]. \tag{17}$$

Therefore, Eq. (15) can be rewritten in the form:

$$\dot{\boldsymbol{\phi}}_i = \mathbf{H}\dot{\mathbf{q}} \tag{18}$$

$$\mathbf{H} = \Phi \lfloor a_{ji}^k + a_{ji}^0 f_k \rfloor_{jk} \tag{19}$$

The present method uses Eq. (18) for determining the joint angles in order to rotate the major axis of DME toward target point of the end-effector motion.

2.3 Joint Velocity and Subtask Approach

If we have redundant degrees of freedom in the manipulator motion, it is possible to design the joint trajectory to perform multiple tasks. Let the task with highest priority be to move the hand toward the target point and let the secondary task be to rotate the major axis of DME toward the direction of hand motion. The conditions corresponding to these two tasks are respectively given by the following equations:

$$\mathbf{v}_d = \mathbf{J}\dot{\mathbf{q}}_d \tag{20}$$

$$\dot{\boldsymbol{\phi}}_{id} = \mathbf{H}\dot{\mathbf{q}}_d \tag{21}$$

Then, the subtask approach provides the desired joint angular velocity that performs both tasks simultaneously:

$$\dot{\mathbf{q}}_d = \mathbf{J}^+\mathbf{v}_d + (\mathbf{I} - \mathbf{J}^+\mathbf{J})\tilde{\mathbf{H}}^+(\dot{\boldsymbol{\phi}}_{id} - \mathbf{H}\mathbf{J}^+\mathbf{v}_d) \tag{22}$$

$$\tilde{\mathbf{H}} = \mathbf{H}(\mathbf{I} - \mathbf{J}^+\mathbf{J}) \tag{23}$$

3. Control Algorithm

A non-linear feedback compensation is employed and a new control variable $\mathbf{u}_q$ is introduced as:

$$\boldsymbol{\tau} = \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{M}(\mathbf{q})\mathbf{u}_q\ , \tag{24}$$

Then, one obtains a linear and uncoupled system with respect to joint variables.

$$\ddot{\mathbf{q}} = \mathbf{u}_q \tag{25}$$

The new input $\mathbf{u}_q$ can be determined by using a servo system to compensate model errors and disturbances:

$$\mathbf{u}_q = \ddot{\mathbf{q}}_d + \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) \tag{26}$$

$\mathbf{K}_v, \mathbf{K}_p$: feedback gains

The control torque can be obtained by using Newton-Euler Method and an efficient computation algorithm for inverse dynamics. The control algorithm is summarized as follows:

Step 1

Compute the desired joint angular velocity $\dot{\mathbf{q}}_d$ that moves the end-effector from present location to the target point and rotate the DME to align the major axis with the direction of hand motion.

$$\dot{\mathbf{q}}_d = \mathbf{J}^+\mathbf{v}_d + (\mathbf{I} - \mathbf{J}^+\mathbf{J})\widetilde{\mathbf{H}}^+(\dot{\boldsymbol{\phi}}_{id} - \mathbf{H}\mathbf{J}^+\mathbf{v}_d) \quad (27)$$

Step 2

Compute the desired joint angular acceleration and joint angle from the desired joint angular velocity $\dot{\mathbf{q}}_d$ (Step 1) and present joint angle $\mathbf{q}(t)$, joint angular velocity $\dot{\mathbf{q}}(t)$ and joint angular acceleration $\ddot{\mathbf{q}}(t)$.

Step 3

Compute the present joint control torque by using present $\dot{\mathbf{q}}(t)$, $\mathbf{q}(t)$ and desired joint angular acceleration $\ddot{\mathbf{q}}_d$, desired joint angular velocity $\dot{\mathbf{q}}_d$, and desired joint angle $\mathbf{q}_d$:

$$\mathbf{M}\mathbf{u}_q + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} \quad (28)$$

$$\mathbf{u}_q = \ddot{\mathbf{q}}_d + \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) \quad (29)$$

Step 4

Compute joint angular acceleration at time $(t + \Delta t)$ for control torque $\boldsymbol{\tau}$ (Step 3):

$$\ddot{\mathbf{q}}(t + \Delta t) = \mathbf{M}^{-1}\{\boldsymbol{\tau} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})\} \quad (30)$$

Step 5

Compute joint angular velocity $\dot{\mathbf{q}}(t + \Delta t)$ and joint angle $\mathbf{q}(t + \Delta t)$ by numerical integration of $\ddot{\mathbf{q}}(t + \Delta t)$.

Step 6

Compute the velocity and position of the end-effector at time $(t + \Delta t)$. If the end-effector position is not close to the target location, then repeat from Step 1 and continue these steps until the end-effector position error becomes smaller than a given threshold value..

4. Numerical Simulations

A numerical example is given for the 2-D model of a 3-link space manipulator to show the capability of the present control algorithm (Fig. 1). The configuration parameters are given as follows:

Mass [Kg]:

$M_0 : M_1 : M_2 : M_3$ =2000: 50: 50: 50

Link length [m]:

$l_0 : l_1 : l_2 : l_3$ =3.5: 2.5: 2.5: 2.5

Moment of inertia [Kgm^2]

$I_0 : I_1 : I_2 : I_3$ =2040: 26: 26: 26

4.1 Local Path Planning using DME

Results of the numerical simulation for the present local path planning method are shown in Figs. 2-4, where the task is the end-effector motion in 10 seconds following to given position and velocity profiles. Figure 2 shows the motion of the manipulator links in x-y plane. The trajectories of the joint angle and control torque are shown in Fig. 3 and Fig. 4, respectively. As shown in Fig. 9, the deviation angle between the major axis of the DME and the direction of the end-effector' motion is decreased with the manipulator motion.

4.2 Global Path Planning Using Optimal Control

The same manipulation task has been formulated as the optimal control problem for minimizing the integrated sum of the total joint torque and the end-effector trajectory tracking error. The Sequential Conjugate Gradient and Restoration Algorithm

(SCGRA) [4] is employed to solve the two-point boundary value problem iteratively. The SCGRA consists of two phases, i.e., the conjugate gradient phase to improve the cost function by gradient search and the restoration phase to reduce the errors in the constraining conditions, and obtains the optimal solution by repeating the two phases alternately. The method provides the global optimum solution but it is not fitted to on-line operation with system uncertainties.

The results of numerical simulation for the optimal control with SCGR algorithm are shown in Figs. 5-7. Figures 8 compares the present DME based control with the optimal control (SCGRA) as to the total joint torque. The accumulated total torque required in the motion is 11.26Nm-sec with the present DME based control method, and 4.82Nm-sec with the optimal control, whereas it was 19.80 Nm-sec for the conventional norm (of joint angles) minimization control [5]. The deviation angle of the DME major axis to the target direction is shown for both control methods (DME/SCGRA) in Fig. 9. It is observed that the optimal control also rotates the DME major axis to align with the direction of end-effector motion. This fact supports that the present DME based control method is reasonable in reducing the joint torque effectively.

5. Conclusions

The use of dynamic manipulability ellipsoid (DME) is proposed for path planning of space manipulators to reduce the required joint torque. The result of numerical simulations shows that the present method is useful for on-line path planning and control of space manipulators in reducing joint torque. Further study is needed for investigating the effectiveness of the present method in various tasks and for different manipulator configurations.

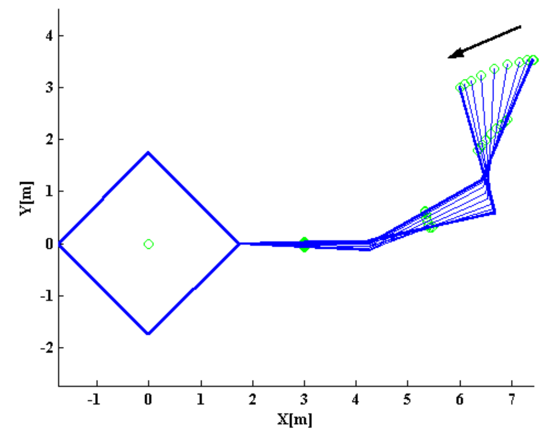

Fig. 2 Manipulator motion (DME)

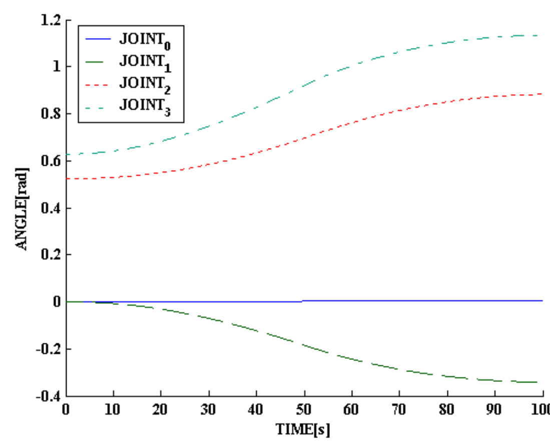

Fig. 3 Trajectories of joint angle (DME)

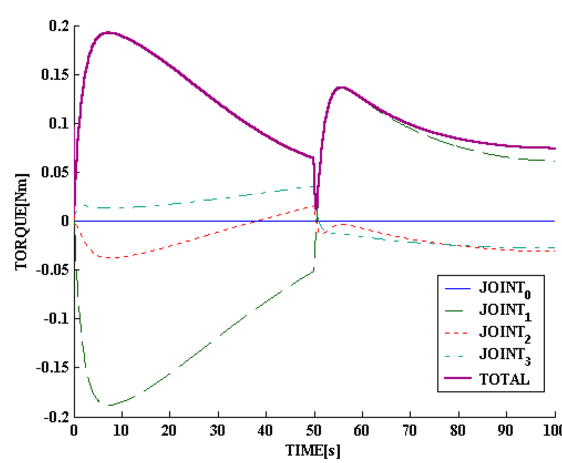

Fig. 4 Trajectories of joint torque (DME)

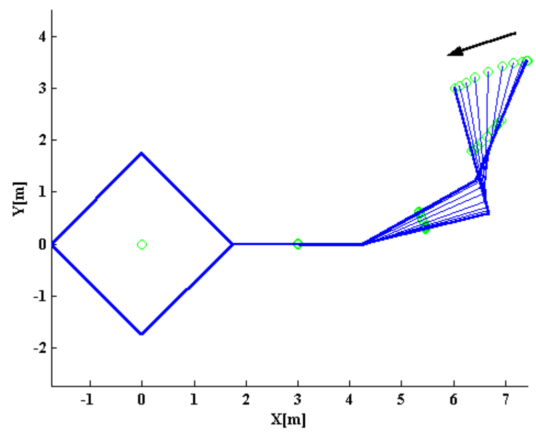

Fig. 5 Manipulator motion (SCGRA)

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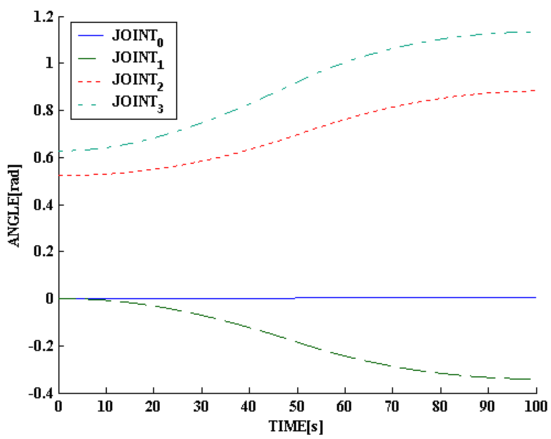

Fig. 6 Trajectory of joint angles (SCGRA)

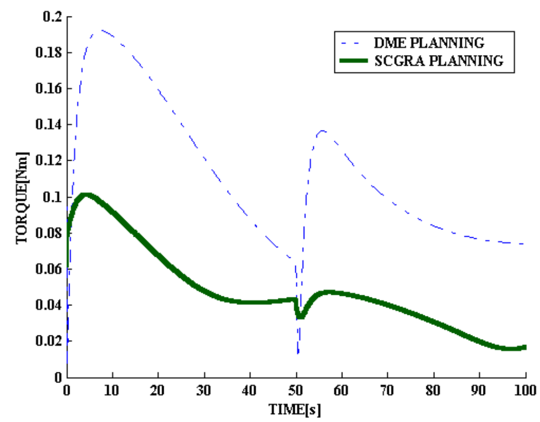

Fig. 8 Total joint torque (DME/SCGRA)

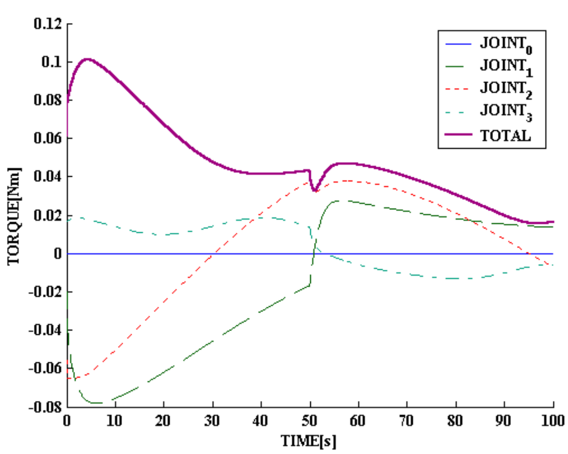

Fig. 7 Trajectories of joint torque (SCGRA)

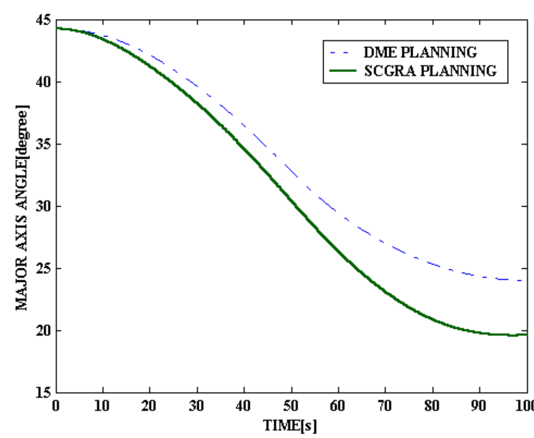

Fig. 9 Rotation of DME major axis (DME/SCGRA)