

DIAGNOSIS METHOD FOR SPACECRAFT USING DYNAMIC BAYESIAN NETWORKS

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ABSTRACT

Development of sophisticated anomaly detection and diagnosis methods for spacecraft is one of the important problems in space system operation. In this study, we propose a diagnosis method for spacecraft using probabilistic reasoning and statistical learning with Dynamic Bayesian Networks (DBNs). In this method, the DBNs are initially from prior-knowledge, then modified or partly re-constructed by statistical learning with operation data, as a result adaptable and in-depth diagnosis is performed by probabilistic reasoning using the DBNs. The proposed method was applied to the telemetry data that simulates the malfunction of thrusters in rendezvous maneuver of spacecraft, and the effectiveness of the method was confirmed.

Key words: Diagnosis, Fault Detection, Dynamic Bayesian Networks, Probabilistic Reasoning, Statistical Learning.

1. INTRODUCTION

A sophisticated diagnosis method automating a series of tasks which contains anomaly detection, evaluation of its dangerousness and appropriate correspondence is indispensable for the safety of space development. Such tasks are almost dealt with by manual operations in the present circumstances though spacecraft such as a satellite and a rocket is a highly automated system. However larger-scale and more complicated a space system is, more knowledge and more tasks are required for its operation. So the established operation methods relying on manpower become difficult to guarantee complete reliability. Actually, the cases that a man slip over the signs of systems' fault have increased. For this reason, it is indispensable to make men concentrate on the most important decisions by automating these tasks with computers.

Several expert systems and model-based approaches including qualitative diagnosis have been developed as the intelligent diagnosis method using computers. Expert systems diagnose by classifying phenomena with the experts' empirical knowledge in the form of "if-then" production rules (Tallo 92; Ciceri 94; Nishigori 01). And in model-based diagnosis, mathematical or qualitative models are utilized to simulate the system behavior and check the validity of the actual sensor values (Monsterman 99; Struss 03). An example model-based diagnosis system is Livingstone (Williams 96), which flew on the deep Space One spacecraft as a part of the Remote Agent Experiment (Muscettola 98). And Recently the diagnosis method using Particle Filter for a planet rover is also presented (Deaden 04). These approaches using prior-knowledge for diagnosis have high cause-investigation power. However there is the significant problem that the accuracy of the embedded knowledge representing the model or the rules effects directly the possibility and the precision of diagnosis for spacecraft because the past fault cases of spacecraft are absolutely few and spacecraft are in the environment containing a lot of uncertainty. On the other hand, in the field of other than space, diagnosis methods putting emphasis on data are proposed which detect the deviances of systems' behavior by using various statistical techniques or analysing the trends of data (Markou 03; Venkatasubramanian 03). These methods, though don't reach the approach putting emphasis on knowledge in the point of accuracy or details of diagnosis, need little knowledge beforehand and enable to diagnose in adaptable way.

In other words, the former approach enables "narrow and deep" diagnosis by using diagnosis model deductively and the latter "wide and shallow" by acquiring one inductively. However, these two polar approaches have the difficulty for covering the other's property and there are few researches fusing both approaches. So we propose a diagnosis method using Dynamic Bayesian Networks for the purpose of the spacecraft diagnosis system which has the wide and deep diagnosis ability by using both knowledge

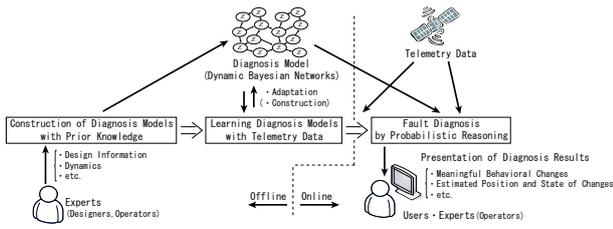


Figure 1. Abstract of the proposed diagnosis method

and data. DBNs have rich expressiveness and understandability as containing Kalman Filter Models (KFMs) and Hidden Markov Models (HMMs), and are the models suitable for representing dynamical systems such as spacecraft. In addition, there are several effective algorithms for statistical learning and probabilistic reasoning for DBNs, the diagnosis method using knowledge and data in natural way are possible to be formed.

The remainder of this paper is organized as follows. The diagnosis method using DBNs is suggested and explanations are given for the each processes in this method. And the superiority of the proposed method is illustrated by applying to the telemetry data that simulates the malfunctioning of thrusters in rendezvous maneuver of spacecraft. Finally, a conclusion is given.

2. APPROACH

The proposed diagnosis method uses Dynamic Bayesian Networks as diagnosis models and consists of following three processes as shown in Figure 1.

1. Construct a diagnosis model with prior-knowledge
2. Learn the model with the stored data
3. Diagnose by probabilistic reasoning with the model

First, in (1), the diagnosis model is constructed from physical knowledge, such as dynamics and design information, and experts' knowledge. Then, in (2), the model is corrected or partly acquired by statistical learning with operation or experiment data obtained beforehand. By this process, it becomes possible to re-construct the model of the parts which are difficult to model with prior-knowledge or correct the error according to modeling. And in (3), anomaly detection and investigation of its cause or position are performed by probabilistic reasoning using the model.

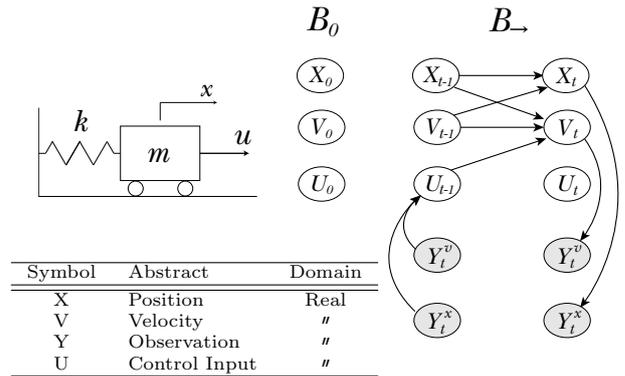


Figure 2. The DBN representing a single-degree-of-freedom system

After this, we first give a summary of DBNs in Subsection 2.1. And in the following three subsections, the explanations about each processes mentioned above are given sequentially.

2.1. Dynamic Bayesian Networks

In this study, we use the probabilistic graphical models called Dynamic Bayesian Networks (DBNs) as diagnosis models¹. A DBN is the model which extend a Bayesian Network (BN) (alternatively Probabilistic Network, Belief Network or Causal Model)² for applying to a dynamical system and is a general state-space-model containing Kalman Filter Model (KFM) or Hidden Markov Model (HMM). A BN is the model possible to deal with uncertain information which is relatively difficult to deal with for computers and studied actively in the field including Artificial Intelligence(Haddawy 99). Its applications reach a wide area from user modeling (Horvitz 98), diagnosis (Jensen 01) to gene analysis (Segal 03). And a DBN, for which the research on various algorithms advances in recent years, is applied to voice recognition etc. (Nefian 02).

A BN consists of the graphical structure G containing the nodes which mean variables and the arcs which represent the correlation between variables and the conditional probability (and its parameters Θ) describing its correlation quantitatively. And a DBN consists of two BNs: BN_0 which represents the prior-distribution $\Pr(\mathbf{X}_0)$ and BN_+ , the transition probability $\Pr(\mathbf{X}_t|\mathbf{X}_{t-1})$. As a brief example, we consider the DBN to estimate the states of a single-degree-of-freedom system represented as follows.

$$\ddot{x} = -k_1 \cdot x + u, \quad u = k_2^t y \quad (1)$$

¹(Murphy 02) is a famous doctoral thesis about DBNs.

²We recommend (Pearl 88) as the most famous traditional book, and (Jensen 01; Cowell 99; Pearl 00) as other famous ones

The graphical structure of the DBN representing this system is shown in Figure 2. The observable variables are represented by the gray nodes and the not-observable variables are the white nodes. And the conditional probabilities are given to each variables, for example about v_t , the following conditional probability can be given.

$$\Pr(V_t|X_{t-1}, V_{t-1}, U_{t-1}) = \mathcal{N}(V_{t-1} + \Delta t(-k_1 X_{t-1} + U_{t-1}), \Sigma) \quad (2)$$

where Σ means the covariance matrix for the normal distribution. In this case, the DBN contains only the continuous variables, however qualitative variables is also possible to be incorporated.

2.2. Constructing DBNs with Prior-Knowledge

The first step is the construction of DBNs and needs the following procedure.

1. Select variables in the model and knowledge used in diagnosis
2. Initialize the graphical structure and the parameters of the DBN using diagnosis knowledge

First, the set of variables represented with the model needs to be selected from the purpose of diagnosis and the users' understanding about spacecraft. In the case of the example described above, if a user needs to consider the transition of the friction between the mass point and floor, the variable representing the friction is added. Then the structure and parameters are setted in the following way from the differential equations representing the process of spacecraft or the experts' empirical rules.

- Physical knowledge represented as a differential equation

$$\dot{\mathbf{x}} = f(\mathbf{x}) \Rightarrow \Pr(X_t|X_{t-1}) = \mathcal{N}(x_{t-1} + \Delta t \cdot f(x_{t-1}), \Sigma)$$

- Experts' empirical knowledge represented as a production rule

$$(\text{if } A = i \text{ then } B = j) \Rightarrow \Pr(A|B) = M(i, j)$$

where Δt means the time-lag and M is the matrix representing the probabilities of each cases. Here though the parameters including variances need to be given, users not necessarily set detailed these parameters because the correctness by statistical learning with data is possible later.

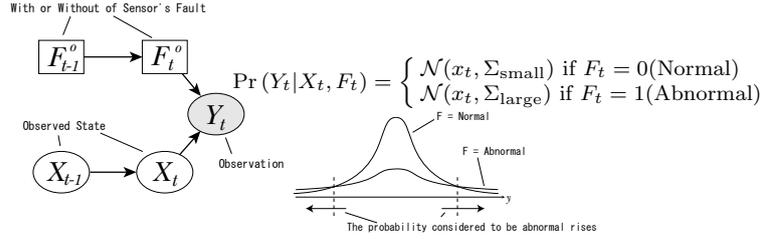


Figure 3. A DBN for diagnosis of sensors

2.3. Learning DBNs with Data

The DBN constructed from prior-knowledge is corrected or partly re-constructed by statistical learning with the stored data as come close to the actual behavior of spacecraft. The learning means the maximization of log-likelihood L represented as following equation.

$$L = \sum_{t=1}^{\tau} \log \Pr(\mathbf{y}_t|G, \Theta) \quad (3)$$

where y_t means the observation at every time t , G and Θ mean the graphical structure and the parameters of the DBN respectively as described above. A log-likelihood is the logarithm of the conditional probability of the observation when given the model, and is the measure how the model explains the observations obtained. For maximizing (3) of the model having some hidden variables, EM Algorithms (Expectation Maximization Algorithm)(Bilmes 98) are generally applied. In our method, the graphical structure and the parameters of a DBN are learned to fit the actual behavior of spacecraft using EM Algorithms with the stored data (Murphy 98; Geiger 94; Friedman 97).

2.4. Diagnosis by Probabilistic Reasoning with DBNs

Our proposed method diagnoses by detecting statistical deviances every variables with on-line probabilistic reasoning using the DBN obtained.

A purpose of probabilistic inference targeting dynamical systems is to calculate the marginal probability described as the following equation at every time.

$$\Pr(X_t|y_{1:\tau}) = c \Pr(y_{t+1:\tau}|X_t, y_{1:t}) \Pr(X_t, y_{1:t}) = c \Pr(y_{t+1:\tau}|X_t) \Pr(X_t|y_{1:t}) \quad (4)$$

where \mathbf{X}_t means the hidden variables at time t , $y_{1:\tau}$ the observation by now, and c the normalized term.

In our method, the mixture of the normal- and abnormal-distribution is attached to the objective

variable, and the transition of the each mixture parameters are calculated for the purpose of detecting statistical deviances every variables from the normal states. For example in the case of a sensor, in addition of the observed variable, the discrete variable whose domain is "normal" and "abnormal" is added as the condition of the observation variable. Then the distribution that has the mean equals to and the variance larger than the normal one is attached as the distribution of $F = (\text{abnormal})$. As a result, the larger the deviance of the observation from the normal behavior, the larger the probability of $F = (\text{abnormal})$ becomes.

The calculation of probabilistic reasoning on DBNs can be generally performed by applying the junction tree algorithm (Lauritzen 99). But in the case that discrete variables have no continuous parent variables like our case, it is impossible to perform the exact computation of probabilistic reasoning because the number of situations increases in exponential with the passage of time. So our proposed method uses the approximate inference algorithm called Rao-Blackwellised Particle Filter (RBPF)(Doucet 00; Doucet 01).

3. RESULTS

In this section, our proposed diagnosis method is applied to the telemetry data that simulates the malfunction of thrusters in rendezvous maneuver of spacecraft³. The results are presented and the consideration is given.

3.1. Problem Set-up

The spacecraft in rendezvous maneuver process controls its attitude and transition using the fourteen thrusters. The data consists of the 26 series including the commands of 16 levels to these fourteen thrusters (14 series) and the states of the spacecraft (position x , velocity v , euler angular θ and angular rate ω) (12 series). The sampling rate of the data is 125 milli-seconds and the simulations are for 1000 seconds. And the mass properties (the weight and the moment of inertia of spacecraft) and the thrusters' properties (the normal thrusts and the location of thrusters) are given beforehand, however the other information such as the orbital characteristics, the control strategy, the sensors' properties and so on are not given. So the nonlinear filters which need to be modelled accurately beforehand (EKF or PF) are difficult to be applied.

³The data is presented from Japan Aerospace Exploration Agency (JAXA)

Table 1. Variables in the DBN

Symbol	Description	Domain
X	Position	Real
V	Velocity	"
W	Angular Rate	"
Θ	Euler Angular	"
T	Thrust	"
Y	Observation	"
C	Command	1 ~ 16(Integer)
F	Presence of Fault	0(Normal),1(Abnormal)

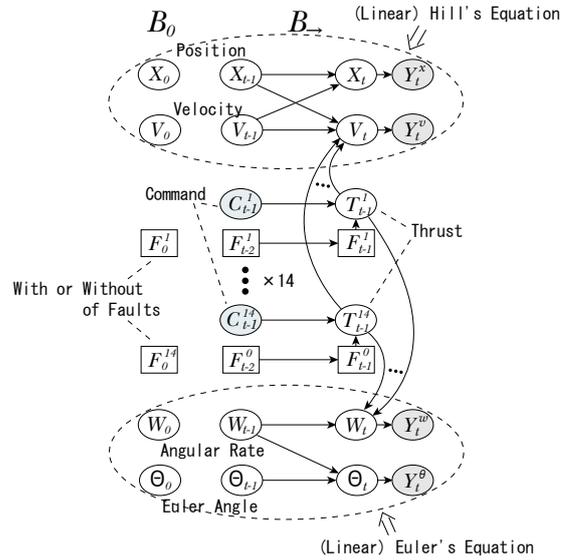


Figure 4. Graphical Structure Used in the Experiment

3.2. Construction of Diagnosis Model

For this experiment, we constructed the DBN that has the graphical structure shown in Figure 4 according to the procedures described below.

First, because of the purpose of the diagnosis described above, we added the continuous variables that mean the thrusts of each thrusters and the discrete variables that mean the presences of anomalies and selected the set of variables shown in Table 1. Then the following linearized equations that mean the attitude and transition dynamics on orbit are used as prior-knowledge.

$$\dot{\theta} = [\omega_1 + n\theta_3, \omega_2 + n, \omega_3 - n\theta_1]^T \quad (5)$$

$$\begin{aligned} J_1\dot{\omega}_1 + n(J_2 - J_3)\omega_3 &= M_1 \\ J_2\dot{\omega}_2 &= M_2 \\ J_3\dot{\omega}_3 - n(J_1 - J_2)\omega_1 &= M_3 \end{aligned} \quad (6)$$

Table 2. Evaluated Diagnosis Models

model	structure	prior-knowledge parameter(initial value)	learning
1	figure 4	from eq. (5) - (7) and prior	no-learning
2	figure 4	from eq. (5) - (7) and prior	parameter learning
3	figure 4	random	parameter learning
4	random	random	structure learning

$$\begin{aligned}
 \ddot{x}_1 &= 2n\dot{x}_3 + T_1 \\
 \ddot{x}_2 &= -n^2x_2 + T_2 \\
 \ddot{x}_3 &= 3n^2x_3 - 2n\dot{x}_1 + T_3
 \end{aligned} \tag{7}$$

where n means the mean motion of circular orbit and J means the moment of inertia of body, whose accurate values were not given beforehand. In this experiment, only physical knowledge above is used, however qualitative knowledge such as experts' empirical rules which are represented by discrete variables are possible to be incorporated.

The procedure for constructing the model is followed. Here, the values of each variables at every discrete time t are represented as $*[t]$, and its time-lag as Δt .

1. Make each differential equations differences. For example about θ_1 , the following equation is obtained from Equation (5).

$$\theta_1[t] = \theta_1[t-1] + \Delta t \cdot (\omega_1[t-1] + n\theta_3[t-1])$$

2. Set the graphical structure and the parameter of the DBN using the difference equations obtained in Step 1 and the other prior-knowledge (for example, the thrust $T = \text{normal thrust} \times C/16$ in nominal phase). About $\Theta_1[t]$, for example, $\Theta_1[t-1]$, $\Theta_3[t-1]$ and $\Omega_1[t-1]$ are selected as parent variables and the following conditional probability are given.

$$\begin{aligned}
 &\Pr(\Theta_1[t]|\Theta_1[t-1], \Theta_3[t-1], \Omega_1[t-1]) \\
 &= \mathcal{N}(\Theta_1[t]; \Theta_1[t-1] + \Delta t(\Omega_1[t-1] + n\Theta_3[t-1]), \Sigma)
 \end{aligned}$$

3. Using nominal operation data obtained beforehand, correct the graphical structure and the parameters of the DBN with statistical learning described in Sub-section 2.3.

3.3. Learning Diagnosis Model

The transition of the log-likelihood in learning process in the case of modelling according to the procedure described in Sub-section 3.2 is shown in Figure 5 (Model 2 in the figure), as well as the case of modeling with no prior-knowledge (Model 4) and the case of modeling with the graphical structure and random parameters. Also the Model1 in the figure is the case that no-learning is done (Table 2). From

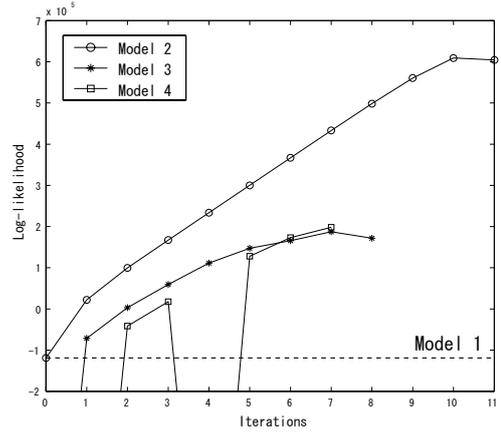


Figure 5. Transition of Log-Likelihood in Learning

Figure 5, we can make certain that the error in the model and the uncertainty of the observation and so on are corrected to adapt to the actual behavior by learning. And from the comparison of Model 2 and Model 3,4, because Model 3 and 4 fall into the local solutions, we can make certain that prior-knowledge is important to acquiring the better model.

3.4. Diagnosis Results and Discussion

In this sub-section, the results when applying our proposed diagnosis method to the 3 fault cases using the DBN obtained with the procedure described above (Model 2) are shown and the perspective about our method are given.

3.4.1. Simple Case

In the first fault case, the thrust of Thruster 4 decreases linearly for 60 seconds from 250 seconds. In this case, the result of the Model1 in Table 2, that is the model not learned, is also shown for comparison.

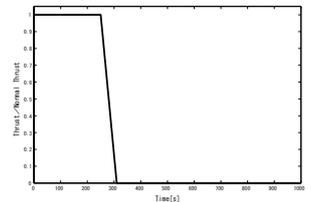


Figure 6. Thrust Decrease of Thruster 4 in the Fault Case 1

First, Figure 7 shows the transition of each thrusters' abnormal probability, that is the probability that each F_i in Figure 4 becomes 1. In these figures, the red lines show the case of the learned model and the blue ones the not learned model. The figure shows that our method using the learned model succeeded in detecting rapidly the anomaly of spacecraft as the values of abnormal probabilities of Thruster 4, 5, 7

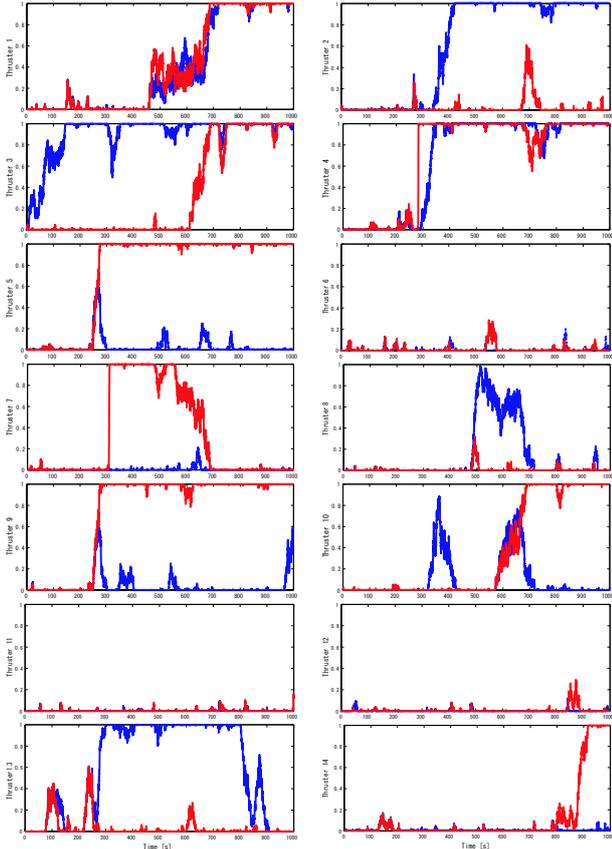


Figure 7. The transition of each thrusters' anomaly probabilities (Fault Case 1)

and 9 change into 1 just after 250 seconds. However the result also shows fail to specify Thruster 4 in which the fault actually occurred, this is probably because of the limit of the states that the observation acquired can distinguish, that is because the observability of the system is not complete. On the other hand, the model with no-learning not succeeded in detecting appropriately as some of the fault probability go up when the actual fault has not happened.

And Figure 8 and 9 show the transition of the thrusts estimated from the observation, that is the mean values of each T_i in Figure 4 at every time. From Figure 8, the situation of the abnormality of Thruster 4 shown in Figure 6 is roughly reproduced when using the learned model. However in the case of using the not learned model (Figure 9), the transitions are random and it is difficult to read some tendencies.

This result shows that the learning process functions well in our proposed method. However at this time, the good results relevant to the structure learning are not obtained.

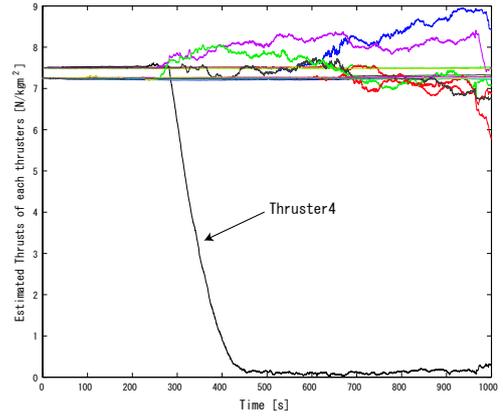


Figure 8. Transition of estimated thrusts (mean values) of each thrusters (fault case1, Model2)

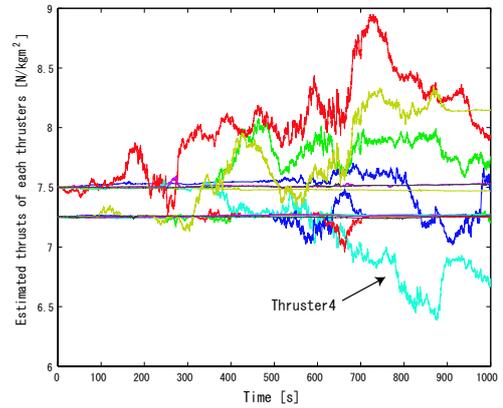


Figure 9. Transition of estimated thrusts (mean values) of each thrusters (fault case1, Model1)

3.4.2. Complex Case

In the actual fault of a thruster, more complicated changes occur such as the gradual decrease of thrust or the repeat of temporal malfunction and recovery. So the results when applying to the fault cases in which the ratio (thrust / normal thrust) of Thruster 9 repeats between 0 and 1 each 60 seconds from 250 seconds as Figure 10 (fault case 2) and the thrust of thruster 9 changes on the signature wave at 60 cycles of the second from 250 seconds as Figure 11 (fault case 3) are shown. For each fault cases, Figure 12 and 14 show the transitions of the abnormal probabilities and Figure 13 and 15 the transitions of the estimated thrusts of each thrusters. About Figure 12 and 14, the graphs whose values keep around 0 almost all region are removed. From Figure 12 and 14, our proposed method, though failed to specify the actual fault thruster as well as the fault case 1, succeeded in rapidly detecting the changes of spacecraft in spite of the complicated fault. However for Figure 13 and 15, though the value of Thruster 9 which actually has

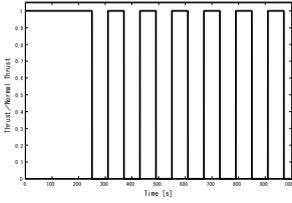


Figure 10. Thrust Decrease of Thruster 9 in the Fault Case 2

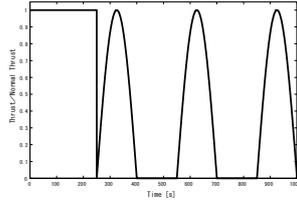


Figure 11. Thrust Decrease of Thruster 9 in the Fault Case 3

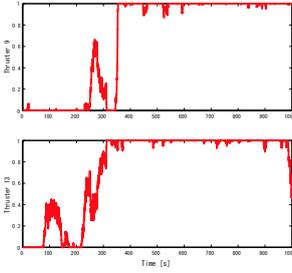


Figure 12. The transition of each thrusters' anomaly probabilities (Fault Case 2)

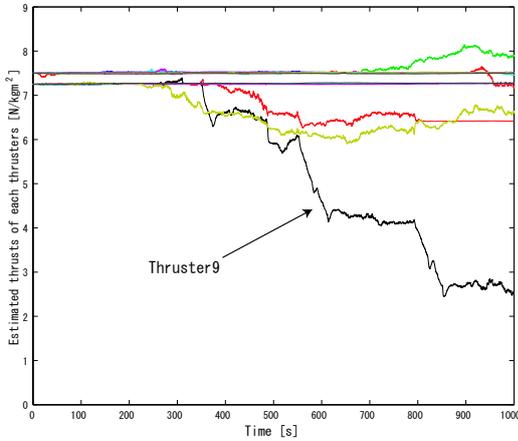


Figure 13. Transition of estimated thrusts (mean values) of each thrusters (fault case 2)

fault, those are considerably different from Figure 10 and 11. Like this it is difficult to follow the large changes of value from the limit of the observability and the accuracy of the model.

4. CONCLUSION

In this paper, we proposed the diagnosis method for spacecraft using DBNs which fuses and uses knowledge and data in the natural way. It has been shown that this method can detect anomaly even in complicated fault cases and make a short list of the fault positions. However we found the problems that it is difficult to specify the fault position completely or reproduce the situation of faults because of in-

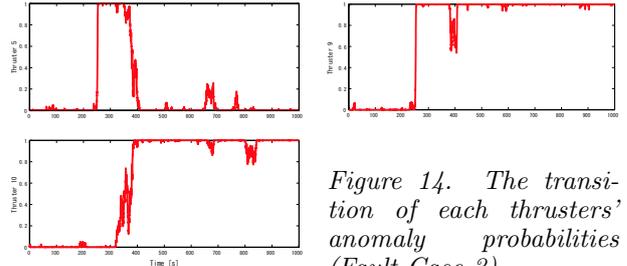


Figure 14. The transition of each thrusters' anomaly probabilities (Fault Case 3)

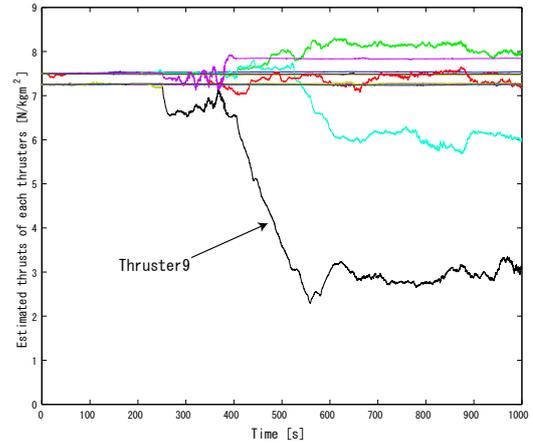


Figure 15. Transition of estimated thrusts (mean values) of each thrusters (fault case 1 3)

completeness of the system's observability and the inaccuracy of the DBNs' representation power. In the future, we are going to examine how to use the various diagnosis knowledge including experts' empirical rules and the representation of the diagnosis model by whom the application to the more practical situation is possible.

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