ABSTRACT

This paper presents an approach to visual navigation for a spacecraft during planetary orbit. Spacecraft orbital parameters are determined autonomously by registration to terrain. To accomplish this, individual camera images are registered to pre-existing maps to determine spacecraft latitude and longitude. Using knowledge of orbital dynamics, the time history of sensed latitudes and longitudes is fit to a physically realizable trajectory. Outlier measurements are rejected using robust estimation techniques. Simulation results using data from the Lunar Reconnaissance Orbiter show that method determines the orbit semi-major axis with an average error less than 6 km for a 500 km altitude lunar orbit.

Key words: orbit determination; vision; robotics.

1. INTRODUCTION

Visual navigation for precise, reliable guidance of crewed and robotic planetary missions offers the opportunity to revolutionize space exploration by improving targeting of landing sites, substantially reducing mission risk, and extending human reach through autonomous missions to distant locations such as the Jovian moons. State-of-practice radio-based navigation exhibits unacceptable latency at long range, relies on interaction with Earth, and cannot achieve the landing accuracy possible through visual means. Autonomous navigation via camera offers an alternative to radio without the mass and power required for traditional altimetry (e.g., RADAR and laser), and without communication to Earth. Visual navigation will enable autonomous spacecraft to land in radio-dark locations like craters and the far sides of planets.

Visual navigation provides three primary advantages over alternatives: accuracy, low latency, and autonomy. During descent, accuracy and latency are important factors in mission success. Far from earth, visual navigation is critical, as communication latency precludes precision maneuvering. Visual navigation for all mission phases will enable fully autonomous operation from launch to landing.

Deep Space 1 demonstrated visual navigation far from planetary surfaces by triangulation to distant asteroids. These observations relied on the asteroid appearance as a single pixel in an imager, reducing measurements to discriminating a point source from black background. Visual navigation during terminal descent has been shown in terrestrial demonstrations to be more accurate than radio navigation and operates with latency independent of distance to Earth. With new high-resolution Mars and Moon imagery, terrain relative navigation can achieve 30-meter localization precision. However, autonomous visual-only navigation is undeveloped for planetary approach, capture, and orbit.

This paper describes Terrain Relative Planetary Orbit Determination (TROD), an approach to visual navigation for orbiting spacecraft. TROD matches terrain appearance to on-line imagery and utilizes knowledge of orbital dynamics to determine position, velocity, and orbital parameters of the spacecraft trajectory. Simulation results using data from the Lunar Reconnaissance Orbiter show that the method determines the semi-major axis with an average error less than 6 km for a 500 km lunar orbit.

1.1. Related Work

Orbit determination has a deep history dating back centuries to early physicists. The related work is deep and it is out of scope to discuss most aspects of orbit determination here. Rather, we focus on spacecraft orbit determination and computer vision approaches to trajectory modeling.

In deep space, camera-based navigation measures angles to distant asteroids to triangulate location [RBS’ 97, BRK’ 02]. These observations rely on the asteroid appearance as a single pixel in an imager, reducing measurements to discriminating a point source from black background. Objects that appear larger than a few pixels introduce significant angular error. The Voyager missions to Jupiter, Saturn and beyond used optical measurements
along with radio to Earth for navigation. Images of the planet’s natural satellites against a star background were used, with knowledge of ephemerides to update a navigation filter run on Earth [CSB83].

Significant work has been done on camera-based terrain relative navigation. While this technique has proved to be quite successful in simulation and sounding rocket testing [TMR06, TMR07, PBC10], the focus has mostly been on the final stages of descent to a planetary surface, not on higher altitude trajectories. Singh and Lim implement an Extended Kalman Filter (EKF) to track spacecraft position and velocity in orbit; they use vision-based measurements, but they also assume that altitude can be obtained from an altimeter with only 12.5m (1σ) error. Such high-end radar is programmatically costly. They use crater-based feature tracking and thus they limit their “experiments to segments of the trajectory that span the higher latitudes where the craters are more clearly visible” [SL08].

In addition to planetary applications, work has also been done on autonomous navigation for landing on asteroids. The Near Earth Asteroid Rendezvous (NEAR) mission to the asteroid Eros was the first to use optical tracking of craters for navigation, but these craters were manually chosen [MC03]. The Hayabusa spacecraft, which conducted a sample return mission to the asteroid Itokawa, used manual tracking of landmarks for localization during descent [KHK09]. Hayabusa also used a long range (50 m to 50 km) LIDAR and a short range laser range finder to determine its altitude [KHK06]. Li presents a feature-based navigation routine for asteroid landing [Li08]. This method provides position and orientation state, although this is also defined relative to the landmarks, not with respect to a global map.

A proposed method for autonomous orbit determination for the NEAR is described in [MC03]. The reliance of this approach on the detection on circular crater features is somewhat limiting. As noted in [SL08], craters are hard to detect when the sun angle is high, since there are no shadows to define the edges of the crater rim. In addition, craters appear different at different altitudes, making them unsuitable for use over the range of altitudes at which a spacecraft localization is needed in planetary landing missions.

2. METHOD

2.1. Overview

TROD works by matching imagery obtained in orbit with orthorectified maps of the planetary surface to determine latitude and longitude over time. Kepler’s laws are then applied to determine orbital parameters.

2.2. Orbit Determination

Kepler’s laws of orbital motion state that the area swept by the line between a body and its orbit center in a given amount of time is equal throughout an orbit [Sid01]. Keplerian orbits are elliptical in shape with a focal point at the barycenter of the system. Consequently, orbits are uniquely determined by several measurements of latitude, longitude, over time as depicted in Fig. 1.

Elliptical orbit of a spacecraft about a planetary body is described by the six classical orbital elements:

- $a$: semi-major axis
- $e$: eccentricity
- $i$: inclination
- $\omega$: argument of periapsis
- $\Omega$: right ascension of the ascending node (RAAN)
- $M$: mean anomaly

The parameters $a$ and $e$ describe the size and shape of the ellipse. The parameters $i$, $\omega$ and $\Omega$ describe the orientation of the ellipse in space, (See Fig. 2). $M$ describes where the spacecraft is in the orbit, although the related parameter $\theta$, or true anomaly, is often used.

Orbit determination is performed in two stages. First, inclination and RAAN are determined by fitting a plane to unit vectors that extend from the orbit center towards the spacecraft. Once the plane has been determined, the noisy unit vectors are projected onto the plane to determine spacecraft angular motion over time. Kepler’s laws describing the relationship between angular motion over time are inverted using non-linear least squares to determine $a$ and $e$. The argument of periapsis is then determined from the other five parameters and the measure-
2.2.1. Fitting the Orbital Plane

As the spacecraft orbits, it images the surface below to determine its latitude, φ, and longitude, λ. The latitude and longitude coordinates are transformed into unit vectors and rotated into the J2000 frame:

\[
\hat{r} = R(t) \hat{r}^f
\]

Where \(\hat{r}^f\) is the unit vector in the moon-fixed frame, \(R(t)\) is the rotation between the moon-fixed frame and latitude measurements and the J2000 reference frame at the time of the first measurement, and \(\hat{r}\) is the unit vector represented in the J2000 frame.

In the J2000 frame, the unit vectors serve as noisy measurements of the orbital plane. The smallest eigenvalue of the covariance matrix of the unit vectors corresponds to the eigenvector representing the normal of the plane. Singular Value Decomposition is used to find the eigenvector and the normal:

\[
\Sigma = \sum_i \hat{r}_i \hat{r}_i^T
\]

\[
[U, S, V^*] = \text{svd}(\Sigma)
\]

\[
\hat{n} = \hat{v}_{m_{\text{min}}}
\]

The sign of \(\hat{n}\) is arbitrary and noise in the points can cause the normal to point in either direction. The direction of \(\hat{n}\) is chosen to agree with the average normalized cross product of adjacent unit vectors. RANSAC is used to reject spurious measurements.

The measurement coordinate system is defined by setting the \(\hat{z}_m\) axis equal to \(\hat{n}\) and aligning the \(\hat{y}_m\) axis with the cross product of \(\hat{n}\) and \(\hat{r}_0\). The \(\hat{x}_m\) axis completes the right-handed coordinate system.

\[
\hat{z}_m = \hat{n}
\]

\[
\hat{y}_m = \hat{n} \times \hat{r}_0
\]

\[
\hat{x}_m = (\hat{n} \times \hat{r}_0) \times \hat{z}
\]

The inclination of the orbit, \(i\), is defined as the angle between the orbital and equatorial planes:

\[
\hat{z}_e = [0, 0, 1]^T
\]

\[
i = \arccos(\hat{z}_m \hat{z}_e)
\]

\(\Omega\) is calculated by taking the cross product of \(\hat{z}_e\) and \(\hat{z}_m\) to get the ascending node line vector, \(\hat{l}\). The ascending node line vector is then projected onto both the \(\hat{x}_m\) and \(\hat{y}_m\) axes to determine \(\Omega\):

\[
l = \hat{z}_e \times \hat{z}_m
\]

\[
\Omega = \arctan2(\hat{y}_m^T l, \hat{x}_m^T l)
\]

2.2.2. Fitting the Keplerian Parameters

Kepler’s laws state that the vector between a satellite and the body which it is orbiting sweeps equal areas in equal time. The period of an orbit is proportional to \(a^{3/2}\), where \(a\) is the semi-major axis. Combining these laws and knowledge of the gravitational constant of the central body, \(\mu\), the angle traveled by the spacecraft in a given time interval can be determined. The equation relating time and angle for an elliptical orbit is [Sid01]:

\[
t(a, e, \theta) = \frac{a^{\frac{3}{2}}}{\sqrt{\mu}} \left[ 2 \arctan \left(\frac{1 - e}{1 + e} \tan \left(\frac{\theta}{2}\right)\right) - \frac{e \sqrt{1 - e^2} \sin \theta}{1 + e \cos \theta} \right]
\]

Where \(t(a, e, \theta)\) is time since periapsis and \(\theta\) is the angle from the periapsis.

The measurement coordinate system is not aligned with the periapsis, so the offset in time and angle between the periapsis and the first measurement must be recovered. Define \(\delta_i\) as the measurement angle in the measurement coordinate frame and \(\alpha_p\) as the offset between the orbital coordinate system and the measurement coordinate system (Figure 1):

\[
\delta_i = \arctan2(\hat{y}_m^T \hat{r}_1, \hat{x}_m^T \hat{r}_1)
\]

\[
\theta_i = \delta_i + \alpha_p
\]

\[
t_p = t(a, e, \alpha_p)
\]
Equation 13 can now be used to define an objective (Equation 17) for optimization. The objective is optimized using non-linear least squares.

\[ J(e, a, \alpha_p) = \left( t_i - t(e, a, \alpha_p) - t(e, a, \tilde{\alpha}_i + \alpha_p) \right)^2 \]  
(17)

The remaining parameters, \( \omega \) and \( M \), are computed directly as follows [Sid01]:

\[ \omega = \text{atan2} \left( \frac{r_{0x}}{\sin(\Omega)}, \frac{r_{0y} \cos(\Omega) + r_{0z} \sin(\Omega)}{\sin(\Omega)} \right) + \alpha_p \]

\[ M_i = (t_i - t_p) \sqrt{\frac{\mu}{a^3}} \]

2.3. Terrain Matching

To determine latitude and longitude of the spacecraft, images are taken in flight and are then registered to prior terrain maps. Terrain matching occurs in three phases: image rectification, coarse correlation, and fine adjustment. Rectification scales and transforms the image to roughly match the scale and orientation of the map. Coarse correlation finds a low-fidelity correspondence between the terrain and the rectified image and accounts for gross errors in initial latitude, longitude, and altitude. Fine adjustment accounts for local variation in terrain within the map and refines the coarse fit.

Rectification is performed using a prior estimate of the spacecraft altitude and orientation. The corners of the image are projected to 2D points on the map using a flat ground assumption. These four point correspondences define a homography that roughly maps camera points to map points. Using the estimated homography, the image is warped to match the orthorectified map. The resulting warped image has the same scale and orientation as the map, and can now be correlated. This process is similar to that used by Trawny, et al. [MTR+09].

The rectified image is downsampled and correlated with a region of interest (ROI) from the map. The ROI is focused on the estimated image center as determined during rectification and is sized to be three times larger than rectified image in map coordinates. This allows for error in the prior location estimate. Due to the size of the maps, correlation is performed in the frequency domain. Fourier transforms of the maps are precomputed to limit on-line computation time, which is essential for running numerous iterations of the routine. The peak of the rough correlation is used to determine an ROI for a refined estimate of location (Fig. 3).

Given a coarse estimate of alignment between the image and the map, a fit is performed to correct for local terrain shape and attitude error between the camera and the map. The 50 strongest Harris corners [HS88] in the rectified image are projected into the map using the inverse camera matrix and the pose estimate from the coarse registration. A small template from the rectified image around the feature is then correlated with a region of interest in the map near the expected location of the feature. This correlation is performed in the spatial domain and the peak of the correlation determines the location of the feature within the map. The set of feature correspondences are then processed using RANSAC to filter spurious measurements and determine the registration between the camera and map (Fig. 4).

3. RESULTS

TROD performance was evaluated in two stages, first with simulated orbit latitude/longitude measurements generated with AGI’s satellite toolkit (STK) [Ana11] and then with latitude/longitude measurements generated from the output of a Lunar rendering engine. With the simulated measurements, the sensitivity of the orbit fit was tested by varying each of the six parameters individually. Then, normally distributed noise was added to each of the simulated orbits to determine the effect of imperfect data. Lastly, the technique was validated by running the visual terrain-matching simulation and then using its output latitude/longitude data as the input for the orbit fit.

All results were analyzed using Circular Error Probabil-
ity (CEP) as the error metric. The CEP is the radius of the sphere centered on the true position which contains a given percentage of the predicted values. The CEP95, for instance, is the spherical radius containing 95% of the predicted results. For all orbits, CEPs of max and mean error were computed by sampling positions along the true and measured orbits at ten second intervals. The distance between the predicted and actual locations was computed and the mean distances were recorded. The CEP was then calculated for the collection of orbits under consideration.

3.1. Performance with Normally Distributed Noise

Orbit determination performance was characterized by generating a set of orbits with a wide range of orbital parameters. Each orbit was sampled regularly in time to determine latitude and longitude relative to the planetary surface. Latitude and longitude were perturbed by adding normally distributed noise, and the fitting algorithm was applied. Because the noise was normally distributed, RANSAC was disabled. Each noise level was evaluated 100 times per orbit. CEP was measured as a function of the six parameters.

All experiments except those with high eccentricity use the parameters in Table 1 as the basis for the orbit and vary one parameter. Experiments with high eccentricity use the basis orbit parameters, except $a = 3550$ km to avoid impacting the surface of the moon at higher eccentricities.

Table 1: Basis Orbit Parameters

<table>
<thead>
<tr>
<th>$a$</th>
<th>$e$</th>
<th>$i$</th>
<th>$\omega$</th>
<th>$\Omega$</th>
<th>$M_0$</th>
<th>noise ($\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2250 km</td>
<td>0.01</td>
<td>45°</td>
<td>45°</td>
<td>45°</td>
<td>45°</td>
<td>0.1°</td>
</tr>
</tbody>
</table>

Figure 5 shows the variation in CEP as $a$ is varied from 1800 km to 3600 km. Noise is held constant at 0.1 degree. At low altitudes, error in $a$ is consistently less than three meters with a CEP95 of 2.6 km with $a$ equal to 1800km. CEPs increase with $a$ and $e$. This is expected, since the fit relies on accurate angular measurements. As the distance from the planet increases, sensitivity to error correspondingly increases. Inclination and RAAN have essentially no effect on position error, as they are determined entirely by the plane fit (Figures 7 and 8). Because $\omega$ and $M$ cause variation in the portion of the orbit that is observed by the fitter, error increases slightly and periodically as $\omega$ and $M$ vary (Figures 9 and 10).

Terrain matching performance is expected to vary as a function of altitude, camera configuration, and off-nominal lighting effects (e.g., lens flare). To characterize the effect of different levels of noise on fit performance, normal distribution noise was varied from 0.0° to 0.5° and CEP was measured. Figure 11 shows that increasing noise causes linearly increasing error in the fit.

### 3.2. Performance with Terrain Matching

Orbit determination performance with terrain matching was evaluated on 15 orbits. The orbits were generated by varying the inclination, periapsis, and mean anomaly of the basis orbit. This produces varied ground tracks over the planetary surface and accounts for variation in matching performance as a function of terrain appearance.

The terrain matching dataset differs from the orbits in section 3.1 in two important ways. First, half of each orbit is in shadow and therefore only half of the measurements from each orbit can be used. Second, due to lengthy rendering times the duration of each orbit is limited to a single full orbit. These two effects significantly reduce the number of measurements available during orbit determination. Figure 12 shows the effect of decreasing the number of measurements available to fit the orbit. Note that although error increases, as more points are collected this noise will decrease and approach the results in Figure 11.

Despite these challenges, orbit determination with visual matching achieves error only twice as high as in the experiments from section 3.1. Typical mean position errors are below 6 km and almost all errors are below 10 km (Table 2).

<table>
<thead>
<tr>
<th>CEP50</th>
<th>CEP75</th>
<th>CEP80</th>
<th>CEP90</th>
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<tbody>
<tr>
<td>5.6</td>
<td>6.1</td>
<td>9.9</td>
<td>18.1</td>
</tr>
</tbody>
</table>

### 4. CONCLUSIONS AND FUTURE WORK

This paper has presented a method for determining orbits by correlating on-orbit imagery with a map. The technique fits the six orbital parameters required to define a Keplerian orbit. Because the technique uses only camera data, it is appropriate for autonomous spacecraft. Results from 500km Lunar orbits demonstrate that this technique is viable.

Better than 1 km accuracy can be achieved by improving the performance of terrain matching. Increasing map resolution and incorporating digital elevation data will significantly enhance matching capability and accomplish this goal.

Kepler’s laws assume point masses, but real planets have lumpy mass distributions. Low orbits are affected by gravity anomalies and high orbits are perturbed by third bodies. An important next step is to incorporate more comprehensive gravity models into the orbit determination algorithm.

Finally, adaptation of TROD for use with Kalman Filtering techniques will enable real-time low-latency orbit determination. This will close the autonomy loop and in
turn enable spacecraft that can capture, orbit, and descend without human intervention.

REFERENCES


Figure 13: Three example orbit fits using terrain matching. Green circles show locations where measurements were taken. Red circles show the match location. Terrain matching performs well in some locations (top, middle) and poorly in others. Significant error is handled by RANSAC enabling orbit determination despite spurious measurements.


