ABSTRACT
The micro satellite TSUBAME is being developed at the Laboratory of Space System in ISAS/JAXA. In TSUBAME, various types of attitude sensors such as Three-Axis-Magnetometer (TAM), Sun sensors, MEMS gyroscopes, Star Tracker system (STTs), Fiber Optic Gyroscopes (FOGs) are available. TSUBAME has four main missions with several different working modes and the most challenged one is a high-speed attitude maneuver using Control Moment Gyros mode. At this mode, high accuracy STTs and FOGs are used as the main attitude sensors. However, to make sure that TSUBAME is able to finish maneuvering process, maximize 90 degrees in at least 15 seconds, and to prevent unexpectedly discontinued available of STTs data, attitude determination system still keeps using TAM and gyroscope data with a separate filter algorithm to estimate another satellite attitude working as a redundancy system. This paper introduces a real-time fine-tuning algorithm applied for above redundancy system to select the optimal process and measurement noise matrices of the Kalman Filters using the estimated attitude from STTs as a truth reference data. After fine-tuning process, the new proposal KFs not only have better performance but also are able to track the changing of disturbance noises or sensor noises. A large amount of numerical simulation for several satellite working conditions is conducted to show the improved performance of this method.

1. INTRODUCTION
TSUBAME is a micro satellite developed by student-leading project. The dimensions of TSUBAME are (H)490 ×(W)490 ×(D)420 mm³ and the total body mass is about 48 kg. TSUBAME is planned to launch in 2012 December and operate on the Sun-synchronous orbit with an altitude of 500 km. One of the main missions of TSUBAME is the real-time gamma-ray bursts (GRBs) observation by using the X-ray detector system which is able to detect GRBs in 180×180 degree field of view. After detect the occurrence of a burst, TSUBAME possesses four Control Moment Gyros (CMGs) which enables a rapid attitude control to start polarimetry observations by a Hard X-ray Compton Polarimeter.

GRBs are flashes of gamma rays associated with extremely energetic explosions that have been observed in distant galaxies. They are the brightest electromagnetic events known to occur in the universe. Bursts can last from ten milliseconds to several minutes; a typical burst lasts 20–40 seconds and its position is unexpected. Therefore, to observer a typical burst, TSUBAME is required to finish pointing and stabilization processes within 15 seconds.

The most well-known filters used for satellite attitude estimation are the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF). EKF is faster and easier to apply but UKF is more robust with initial attitude error, process noise and measurement noise covariance matrices selection [1][2].

Figure 1: Sequence of GRBs observations of TSUBAME

In GRB observation mode, Star Tracker system (STTs), Fiber Optic Gyroscopes (FOGs) and Three-Axis-Magnetometer (TAM) are used as the main sensors to determine satellite attitude. Where, STTs and FOG with EKF algorithm are just enough for estimate satellite attitude and gyro bias; TAM works as a backup sensor for STTs when both star trackers point to the Earth or the taken picture is too blur because of the high rate of satellite attitude for stars detection. The redundant system not only has TAM and FOG data, which is
enough for attitude estimation, but also has quaternion information from STTs. Therefore, a new algorithm is required to design for full use of data in order to maximize the robustness and accuracy of this redundant estimator.

The new algorithm will be developed base on Unscented Kalman Filters [3], a well-known and flight-confirmed algorithm for attitude estimation. In KFs, if the statistics of the process and measurement noise process models do not “match” the actual statistics of these noise processes, the filter estimates will be degraded. The choices of process noise covariance ($Q_k$) and measurement noise covariance ($R_k$) are assumed to be zero mean Gaussian noise with time-varying. The sampling period is denoted by $\Delta t = t_{k+1} - t_k$.

The mission of estimator is to find out the optimal value of next step of a state vector and its covariance matrix based on system model and measurement vector. The idea of UKF is to produce a set of well-chosen sampling points (sigma points) around the current estimated state based on its covariance. Then, propagating these points through the nonlinear map to get an accurate estimation of the mean and covariance of the mapping results. Assume state vector $x$ has current estimated mean $\hat{x}_k$ and covariance $P_k\ddagger$. The set of $2n+1$ sigma points, in form of matrix $\chi_f$, is generated by Eq. (3).

$$\chi_i = \hat{x}_k \pm \gamma \sqrt{P_k\ddagger}$$

where $(\sqrt{P_k\ddagger})_i$ is the $i$th column of matrix square root. The constant $\alpha$ determines the spread of the sigma points around $\hat{x}_k$ and is usually set to a small positive value (e.g., $10^{-4} \leq \alpha \leq 10^{-1}$). The constant $\kappa$ is a secondary scaling parameter which is usually set to 0 or 3-n. The transformed set of sigma points through the nonlinear system are carried out for each point by $\chi_i = f(\chi_i, u_k, k)$

The posterior mean and covariance are given by $\hat{x}_{k+1} = \sum_{i=0}^{2n} W^m_i \chi_{i+1}(i)$

$$P_{k+1} = \sum_{i=0}^{2n} W_i^c (\chi_{i+1}(i) - \hat{x}_{k+1})(\chi_{i+1}(i) - \hat{x}_{k+1})^T + Q_k$$

The weights, $W^m_i$ and $W^c_i$, are calculated using $W^m_0 = \frac{\lambda}{n+\lambda}$

$$W^m_0 = \frac{\lambda}{n+\lambda}$$

$$W^m_0 = W^m_0 + 1 - \alpha^2 + \beta$$

$$W^c_0 = \frac{1}{2(n+\lambda)}$$

$\beta$ is used to incorporate prior knowledge of the distribution of state vector $x$ (for Gaussian distributions, $\beta = 2$ is optimal [10])

The mean measurement and output covariance are given by

$$\hat{y}_{k+1} = \sum_{i=0}^{2n} W_i^c y_{i+1}(i)$$

$$P_{k+1} = \sum_{i=0}^{2n} W_i^c (y_{i+1}(i) - \hat{y}_{k+1})(y_{i+1}(i) - \hat{y}_{k+1})^T$$
where
\[ Y_{k+1}(i) = h(x_{k+1}(i), k) \]  
(12)

The innovation covariance is calculated using
\[ P_{k+1}^w = P_{k+1}^w + R_{k+1} \]  
(13)

Then, cross-correlation matrix is determined by
\[ P_{k+1}^w = \sum_{i=0}^{2k} W_i' \left( x_{k+1}(i) - \hat{x}_{k+1}(i) \right) \left( y_{k+1}(i) - \hat{y}_{k+1}(i) \right)^T \]  
(14)

The Kalman gain matrix is approximated from innovation covariance and cross-correlation matrix using
\[ K_{k+1} = P_{k+1}^w P_{k+1}^{w-1} \]  
(15)

Finally, state and covariance update are calculated by
\[ \hat{x}_{k+1} = \hat{x}_{k+1} + K_{k+1} \left( y_{k+1} - \hat{y}_{k+1} \right) \]  
(16)
\[ P_{k+1} = P_{k+1} - K_{k+1} P_{k+1} K_{k+1}^T \]  
(17)

3. ATTITUDE KINEMATICS AND SENSOR MODELS

3.1 Attitude Kinematics

The quaternion is defined by \( q = \left[ q_3^T \ q_4 \right]^T \) with \( q_3^T = [q_1 \ q_2 \ q_3]^T \) is vector part and \( q_4 \) is the rotation part. The quaternion representation is desirable because of their singularity free property. However, the norm constraint must be maintained. Since a four dimension vector is used to represent three dimensions, the quaternion has a single constraint given by \( q_4 = 1 \). The attitude matrix is calculated as a quadratic function of \( q \), that is
\[ A(q) = \left( q_3^T \ | q_3 \right) I_{3 \times 3} + 2 \rho q_3^T - 2 q_3 \left[ \rho \times \right] \]  
(18)

where \( I_{3 \times 3} \) is a \( 3 \times 3 \) identity matrix and \( \left[ \rho \times \right] \) is the antisymmetric matrix define by [9]
\[ \left[ \rho \times \right] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \]  
(19)

The quaternion kinematic differential equation is defined as below.
\[ \dot{q} = \frac{1}{2} \left( \omega \times \right) q = \frac{1}{2} \Xi(q) \omega \]  
(20)

where \( \Xi(q) = \left[ q_4 I_{3 \times 3} + \left[ \rho \times \right] - \rho \rho^T \right] \) is a \( 4 \times 3 \) matrix, \( \omega \) is the \( 3 \times 1 \) angular velocity vector. In Eq. (21) quaternion multiplication operator [9] “\( \otimes \)” is used, in which \( \tilde{\omega} = \left[ \omega^T \ 0 \right]^T \) is the augmented velocity vector.

3.2 Sensor Models

TSUBAME has two kinds of gyro sensor to measure angular velocity, MEMS gyro and FOG. The general model of gyro can be found in [8].

\[ \omega(t) = \hat{\omega}(t) - \beta(t) - \eta_i(t) \]  
(22)

where \( \hat{\omega}(t) \) is the measured angular velocity, \( \omega(t) \) is the true angular velocity, \( \beta(t) \) is the gyro bias, \( \eta_i(t) \) and \( \eta_i(t) \) are independent zero-mean Gaussian white noise processes with standard deviations \( \sigma_{\omega} \) \( \sigma_{\beta} \). Gyro bias \( \beta(t) = \left[ \beta_1 \ \beta_2 \ \beta_3 \right]^T \) is the unknown value which is considered constant but can vary at each use. TAM and sun sensor are vector observation sensor type and they can be modelled as:
\[ \tilde{y}_i = b_i + v_k = \begin{bmatrix} A(q_{\rho}) \end{bmatrix} + v_k \]  
(23)

where \( v_k \sim \mathcal{N}(0, \mathcal{R}_k) \) is a zero-mean Gaussian white noise with covariance \( \mathcal{R}_k \). \( m \) is the number of vector measurements, and subscript \( k \) denotes measurement is taken at time \( t_k \). \( b_k \) is an vector measurement in spacecraft body frame of \( i^{th} \) sensor, \( q_k \) is attitude quaternion, \( A(q_{\rho}) \) is a correspond attitude direction cosine matrix, \( r_k \) is a corresponding of \( i^{th} \) reference vector model in Earth-fixed frame, and all values are represented at time \( t_k \).

4. FINE-TUNING UKF USING STAR TRACKERS DATA

4.1 Numerical optimal algorithm

Fine-tuning process should include a numerical optimization algorithm performing the mission of finding the best match of process noise covariance (\( Q \)) and measurement noise covariance (\( \mathcal{R} \)) matrices in real-time for each attitude estimation step. The Downhill Simplex method [14] is chosen as the optimal algorithm for tuning process. This is a multidimensional optimization method which uses geometric relationships
to aid in finding function minimums. One distinct advantage of this method is that it does not require taking the derivative of the function being evaluated. Instead it creates its own pseudo-derivative by evaluating enough points to define a derivative for each independent variable of the function. The method revolves around a simplex; a geometric object which contains \( n+1 \) vertices where \( n \) is the number of independent variables. For instance, in the case of a one dimensional optimization the simplex would contain two points and represent a line segment. In the two dimensional optimization, a three pointed triangle would be used, and three dimensions it is a tetrahedron. The function which to be minimized is evaluated at each of the \( N+1 \) simplex vertices.

Each vertex of the simplex will have a function value, and the points with lower values are obviously superior. By evaluating each point the algorithm creates pseudo-derivatives of the function at the location of the simplex. The goal of the function is then to move the simplex away from points of high value. The downhill simplex method can employ several methods for moving the simplex downhill. These methods include reflection, expansion, contraction, and reduction. The method is chosen depends on both data known from the current simplex location as well as data from previous moves in each iteration.

4.2 Parameterization and optimal function

The state vector of estimator is chosen as \( \mathbf{x} = \begin{bmatrix} \Delta p^T & \beta^T \end{bmatrix}^T \) in which, \( \Delta p \) is the local error of spacecraft attitude represented in Modified Rodrigues Parameter (MRP) \([8][9]\) and \( \beta \) is the gyro bias vector. \( Q \) is a \( n \times n \) matrix and \( R \) is a \( 3N \times 3N \) matrix, where: \( n = 6 \) is the dimension of state vector and \( N \) is the number of attitude sensor. As mentioned above, only TAM is used as the main attitude sensor while the quaternion of STTs is used as the reference system for optimal process, so \( N=1 \) in this case. It is too complicate to find the optimal value of all parameters of these matrices; therefore parameterization process is applied to reduce the complexity. Normally in UKF \([2][4]\), \( R \) is defined as

\[
R_0 = \sigma_{\text{TAM}} I_{3 \times 3}
\]

and \( Q \) as

\[
Q_0 = \begin{bmatrix}
\sigma^2_{\Delta t} I_{3 \times 3} + \frac{1}{3} \sigma^2_{\Delta \delta t} I_{3 \times 3} & - \frac{1}{2} \sigma^2_{\Delta t} I_{3 \times 3} \\
- \frac{1}{2} \sigma^2_{\Delta \delta t} I_{3 \times 3} & \sigma^2_{\Delta \delta t} I_{3 \times 3}
\end{bmatrix}
\]

\[
Q = \gamma_Q Q_0 \quad \text{and} \quad R = \gamma_R R_0 \quad \text{are tuned by 2 scaling factors}
\]

\[
\gamma_Q \quad \text{and} \quad \gamma_R \quad \text{by minimize a cost function}
\]

\[
L_{k+1} = \exp \left[ \frac{1}{2} (\delta p_{k+1})^T \left( P_{k+1}^n \right)^{-1} (\delta p_{k+1}) \right]
\]

where

\[
P_{k+1} = \begin{bmatrix}
\begin{array}{c}
p_k^m \\
p_k^s \\
p_k^b \\
p_{k+1}^m \\
p_{k+1}^s \\
p_{k+1}^b
\end{array}
\end{bmatrix}
\]

\[
\delta p_{k+1} \quad \text{is a MRP representation of the propagated error quaternion, which is calculated as} \quad \delta q_{k} = q_{\text{STT}} \otimes q_{\text{ref}}^{-1} \quad \text{at step} \quad k; \quad P_{k+1}^m \in R_{3 \times 3} \quad \text{is sub-matrix of error covariance part matrix} \quad P_{k+1} = R_{6 \times 6} \quad \text{at step} \quad k. \quad \text{Because we only has the reference attitude information from STTs so only attitude part of matrix P is taken into account. In case of STTs is available, the flowchart of proposal algorithm is illustrated in Fig. 2. If STTs is not available the optimal part is rejected and the proposed method converts to the conventional UKF.}
\]

![Figure 2: The proposed algorithm](image-url)
5. NUMERICAL SIMULATION SAMPLES

The proposed algorithm is now applied to several satellite operation scenarios in GRBs observation mode to verify its advantage compared with conventional UKF algorithm. Two sensors used as the main sensors of filter are Three Axis Magnetometers (TAM) - HMR2300 (manufactured by Honeywell Inc.) and Fiber Optic Gyroscope (FOG) - FG34F (manufactured by Japan Aviation Electronics Industry, Ltd). Star Tracker system (STTs) including 2 separate star trackers - STT1LN (manufactured by AXELSPACE Corporation) can directly provide the estimated attitude of satellite in quaternion representation with accuracy less than 30 arcsecond. Main specifications of all sensors are showed in the Tab. 1.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAM</td>
<td>Accuracy (3σ)</td>
<td>520 nT</td>
</tr>
<tr>
<td></td>
<td>Power (W)</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Max. update rate (Hz)</td>
<td>100</td>
</tr>
<tr>
<td>FOG</td>
<td>Accuracy (3σ)</td>
<td>≤ 9 deg/h</td>
</tr>
<tr>
<td></td>
<td>Power (W)</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>Max. update rate (Hz)</td>
<td>&gt;100Hz</td>
</tr>
<tr>
<td>STT</td>
<td>Accuracy (3σ)</td>
<td>30 arcsecond</td>
</tr>
<tr>
<td></td>
<td>Peak Power (W)</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>Max. update Rate (Hz)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Field of View</td>
<td>8x8 deg</td>
</tr>
</tbody>
</table>

The simulation process is setup as Fig. 3. Where, satellite orbit is generated from TLE file based on SGP4 model then Earth magnetic field in ECI frame is calculated by IGRF-11 model. From satellite dynamics and kinematics model, satellite attitude and satellite angular velocity are generated, then through gyro model and TAMs sensor model the omega measured on FOG $\dot{\omega}_i$ and sensor data $y_{i+1}$ are created as the fake sensor data including several type of noises. Star tracker’s data is also simulated by adding the zero-mean white Gaussian noise to the true quaternion.

The first scenario, the satellite is waiting GRBs occur to observe therefore the satellite is stable with low angular rate. The new algorithm – Fine Tuning Unscented Kalman Filter (FTUKF) will be compared with conventional Unscented Kalman Filter (UKF) in this simulation. Total simulation time is 2 hours and sampling frequencies of sensor are both assumed at 2 Hz. The truth initial error of gyro bias is $[30 \ -10 \ 40]^T$ (deg/h) and initial attitude error is $[30 \ -30 \ 27.9]^T$ (deg), both values are unknown.

Initial error covariance matrix is $P_0 = 10^{-3}I_{6x6}$ and initial state vector is $x_0 = [0 \ ... \ 0]^T$. Fig. 4 shows the simulation results, the results plotted here are the norm of 3-axis estimated errors in degree. The estimated error of FTUKF is plotted with continue-red line and UKF is dash-dot-black line. The hidden-green line shows the availability of STTs, when it’s equal to 1 or $10^6$ means that STTs is available. In this simulation, assume that STTs is available from 1000s to 2000s and from 3000s to 3500s. From simulation results we can see that FTUKF will increase its accuracy approached to STT’s level almost right after STT’s available and still better than conventional UKF within about 3000 seconds after STTs is un-available.

In the second scenario, the satellite still works in the same condition with 1st scenario but assume that there are some problems such as the Solar storm shakes Earth magnetic field so the TAM error increases twice times from 520 nT up to 1040 nT and a random TAM bias within 300nT is added in each TAM’s axis. The effect starts from 3000s to 8000s. In this scenario, STTs is turned on for 5 short periods. This simulation is designed to track the response of filters when satellite enters in a shock environment. The simulation is taken in 10 hours and the result is plotted in Fig. 5.

The starting point of bad environment at the point 3000s
is easy to realize because of the increment of estimated error. If STTs is available FTUKF will be not affected by this shock environment also FTUKF has faster recovery to normal condition than UKF. More over, this simulation result shows that FTUKF can be applied with short duration and frequently use of STT, the estimated accuracy and robustness are much more improved but total consumption power is still saved.

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In the last scenario, assume that satellite try to observe a GRB from 3000s to 3015s, also due to high angular rate of satellite, STTs cannot give out its estimated attitude until the satellite become stable again so that STTs is assumed un-available from 3000s to 3180s. From simulation results in Fig. 6 and Fig. 7, we can see at the observation time, because of the effect of high attitude rate, estimated result of both methods are same level however after that, although without star tracker information FTUKF still get better than conventional UKF.

One disadvantage of FTUKF using downhill simplex search is the calculation cost of FTUKF (17.4 ms) is about 6 times of UKF (2.8 ms). The calculation cost is measured for one step of the simulation by Matlab ver. 2008a on a PC. This disadvantage can be improved if faster numerical optimal methods such as gradient-based optimal methods are used. However, it will take a lot of effort to find the derivative of estimated error respect to $Q$&$R$ matrix parameters.

6. CONCLUSION

This paper presented the new method by using Downhill Simplex Search to turn the process noise covariance and measurement noise covariance matrices of conventional Unscented Kalman Filter (UKF) in order to get a new filter named Fine Tuning UKF with higher accuracy and faster convergence speed. By the numerical simulation results, it is proved that there are always exit the optimal value of these matrices to minimize the cost function in each process of filter. The matrices are tuned by a just-enough set of two scaling factors. The good response of new filter is showed in 3 simulation samples with 3 different operation scenarios of TSUBAME in gamma-ray bursts observation mode.

7. ACKNOWLEDGEMENTS

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8. REFERENCE