Experimental Verification of Attitude Determination using Solar Panels and Inclinometer

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Abstract

In this study, we propose a new attitude determination system, which employs the characteristics and geometry of solar panels. First, the sun vector is estimated using data from solar panels including current, voltage, temperature, and the normal vectors of each solar panel. Because these values are obtained using internal sensors, it is easy for rovers to provide redundancy for our proposed system. The normal vectors are used to apply to various shapes of rovers. Second, using the gravity vector obtained from an inclinometer, the attitude of a rover is estimated using a three-axis attitude determination method. The effectiveness of the proposed attitude determination system is verified through experiments that show the proposed system can estimate all the attitude angles (roll, pitch, and yaw) within a few degrees of accuracy, which is adequate for planetary explorations.

1 Introduction

Planetary rovers are being increasingly used in space exploration missions. With their ability to traverse the surface of planets they are capable of in-situ exploration even in areas where landers cannot land, such as steep craters. The National Aeronautics and Space Administration (NASA) sent two rovers, named Mars Exploration Rovers (MER) Spirit and Opportunity, to Mars [1] in 2003. The two rovers subsequently explored the surface of Mars and gathered a wealth of information. In 2011 NASA also sent Curiosity to explore the habitability of Mars [2]. The Japan Aerospace Exploration Agency (JAXA) is currently planning a mission to the moon called the SELENE-2 mission [3]. In this mission, rovers are expected to travel across wide open areas and observe terrain features using onboard scientific instruments.

In planetary exploration missions using rovers, navigation relies on integrated local measurements, such as combinations of inertial sensors, and wheel and visual odometry. These systems are essential for obtaining data on the attitude of rovers that are required for accurate guidance. This system for planetary rovers is called an attitude determination system.

A number of studies have proposed attitude determination systems for rovers. The Mars exploration rovers, Spirit and Opportunity [4], explored the surface of Mars and gathered a wealth of information using an attitude determination system with cameras and inertial sensors. Furgale et al. [5] proposed a system that combined a sun sensor and an inclinometer, and showed its efficiency in experiments.

For future missions using rovers, a redundant attitude determination system is now a requirement. First, because rovers are required to enable long traverse explorations and long mission terms. Additionally a sensor malfunction was reported in the Curiosity mission [6]. A redundant attitude determination system is needed in such cases, but generally rovers have fewer resources for this purpose than spacecraft such as artificial satellites and it is not realistic for rovers to be equipped with identical redundant sensors in case of malfunction. For this reason a redundant attitude determination system should be created by using a different combination of sensors that rovers are already equipped with.

In this study, we propose a new attitude determination system, which combines the characteristics of solar panels, and an inclinometer. The sun vector can be estimated by measuring the current, voltage, and temperature of the solar panels as the electric power generation of each solar panel depends on their arrangement and the position of the sun. By combining this sun vector with the gravity vector obtained using an inclinometer, the attitude of the rover can be estimated. The q method [7] is generally known as a technique for obtaining an attitude from two observation vectors. Santoni et al. [8] proposed an attitude determination system for small spinning spacecraft, which combines the characteristics of solar panels with a magnetometer. This system was aimed at regular octagonal pillar shaped spacecraft, which is a characteristic shape of spinning spacecraft. In this study we employ the minimization of a cost function using the normal vectors of the solar panels of the rovers to apply to various shapes of rovers. The efficiency of our proposed system was validated through experiments.

Our proposed system is useful for planetary rovers. First, it does not require any external sensors. Second, the solar panels necessary for power generation can also be used as sensors for attitude determination.

2 Model

This chapter provides the mathematical framework for the analysis used in this study. We first define the frame of reference and mathematical models for solar panels and noise which has an effect upon an attitude determination system. These noises should be considered and compensated to estimate the attitude precisely.

2.1 Frame of reference

As shown in Fig. 1, the topocentric frame is defined with respect to the local horizon, the Z_t axis is the normal vector to the tangent plane, the Y_t axis is along North and the X_t axis is defined by the right-handed system. In contrast, as shown in Fig. 2, the origin of the rover frame is located at the rover itself, the X_r axis is along the direction of travel, the Z_r axis is vertical to the rover and the Y_r axis is defined by the right-handed system. We define rotation around the X_r axis as roll, the Y_r axis as pitch, and the Z_r axis as yaw.

2.2 Mathematical model

A. Solar panel

The proposed system employs the electric power generated by the solar panels to estimate an attitude of the rover. Therefore solar panel modeling is required as discussed in this section.

There is relationship between the voltage and current of a solar panel, which is called the *I-V* curve. It is possible to store the *I-V* curve in the memory of a rover, but it varies depending on the irradiance and temperature of the solar panel, so it is not realistic to store all the *I-V* curves corresponding to possible irradiance and temperature conditions. Therefore, solar panel modeling is required to obtain the *I-V* curve from sensor values and the ephemeris, for example voltage, temperature and irradiance.

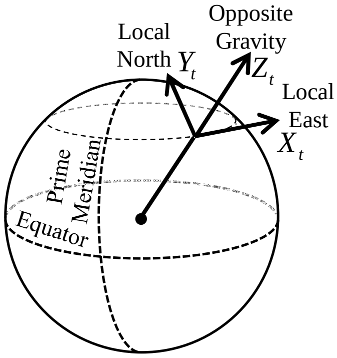

Figure 1. Topocentric coordinate system which is defined with respect to the local horizon

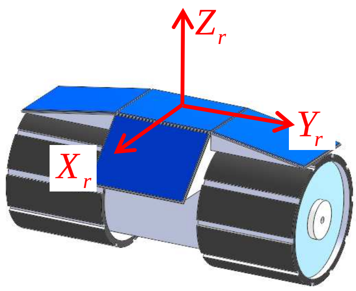

Figure 2. Rover coordinate system in which the origin is at the rover

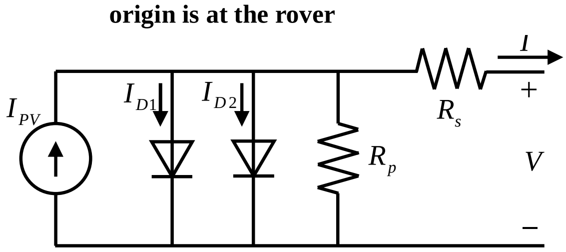

Figure 3. Equivalent circuit of a cell of a solar panel described in the two-diode model [11]

Several solar panel models have been proposed as equivalent circuits [9-11]. In this study, the solar panel model uses a two-diode model [11]. Fig. 3 shows the equivalent circuit describing the two-diode model.

B. Irradiance

Because the experiments were conducted on earth, the irradiance including diffuse radiation should be considered. In this study, we used Erbs model [12]. The details of this model are described in chapter 3.

C. Voltage drop of diode

The solar panels used in our experiments have diodes. Because the voltage is dropped at these diodes depending on the current, the voltage drop should be considered. We approximated this relationship between the current and voltage drop of the diode V_f by a power function of the current I given by:

$$V_f = 0.9826 I^{0.0981} \tag{1}$$

D. Others

The change of the solar constant and the degradation in the solar panels are considered as factors which affect the attitude determination system. The change of the solar constant is approximately 0.1 % [13], and we found the effect to estimate values is less than 0.03 degrees by the numerical simulation. Thus we do not take it into account. The degradation in the solar panels is approximately 0.8 % per year [14], and we found the effect to estimate values is less than 0.1 degrees by the numerical simulation. Thus we do not take it into account either.

3 Attitude determination system

In this study, we propose an attitude determination system that focuses on the characteristics of the solar panels. Fig. 4 shows the conceptual diagram of the proposed system. This method employs the current, voltage and temperature of the solar panels to estimate the sun vector, employs data from an inclinometer to estimate the gravity vector, and then combines the two vectors to estimate the attitude of a rover using the *q* method [7]. In this study, we assumed the position of the rover is known, and that it does not move during attitude estimation.

The direction of the sun is represented by the altitude angle θ_s and azimuth angle α_s. The sun vector $\mathbf{S}_r$ in the rover frame is represented by a vector as given by formula (2), which contains θ_s and α_s. This equation implies that estimating $\mathbf{S}_r$ is the same as estimating θ_s and α_s.

$$\mathbf{S}_r = \left[\cos\theta_s \cos\alpha_s \quad \cos\theta_s \sin\alpha_s \quad \sin\theta_s \right]^T \tag{2}$$

As the current of solar panels are almost proportional to the irradiance, we derive a cost function using this characteristic to estimate the sun vector. Although we need the solar irradiance on inclined surface, generally we can only obtain the global solar irradiance. Therefore we estimate the solar irradiance on inclined surface from the global solar irradiance using Erbs model [12].

The global solar irradiance is represented as sum of the diffuse solar radiation and the direct solar radiation.

$$H = H_d + H_b \tag{3}$$

where H_d and H_b are the diffuse and direct solar radiation, respectively. The ratio of H_d to H_b is derived from:

$if\ H/H_0 \le 0.22$

$$\frac{H_d}{H} = 1.0 - 0.09 \frac{H}{H_0} \tag{4}$$

$if\ 0.22 < H/H_0 \le 0.80$

$$\frac{H_d}{H} = 0.9511 - 0.1604 \frac{H}{H_0} + 4.388 \left(\frac{H}{H_0} \right)^2 - 16.638 \left(\frac{H}{H_0} \right)^3 + 12.366 \left(\frac{H}{H_0} \right)^4 \tag{5}$$

$if\ 0.80 < H/H_0$

$$\frac{H_d}{H} = 0.165 \tag{6}$$

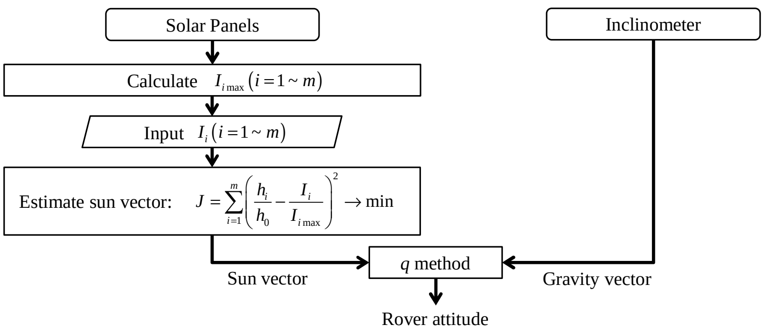

Figure 4. Conceptual diagram of the proposed attitude determination

where H_0 is extraterrestrial solar radiation. The direct, reflected, and diffuse irradiance on inclined surface of each solar panel are given by:

$$h_{bi} = H_b \frac{\cos\theta_i}{\cos\theta_z} \tag{7}$$

$$h_{ri} = Hp\frac{1-\cos\theta_a}{2} \tag{8}$$

$$h_{di} = H_d \frac{1+\cos\theta_a}{2} \tag{9}$$

where θ_z is zenith angle, θ_a is inclination angle of the solar panel, p is albedo, and θ_i is an angle of incidence of each solar panel. θ_i is derived from the definition of the inner product given by:

$$\theta_i = \cos^{-1}\left(\mathbf{n}_i \cdot \mathbf{S}_r\right) \tag{10}$$

where $\mathbf{n}_i$ represents the normal vector of each solar panel. Consequently solar irradiance on the solar panel is given by:

$$h_i = h_{bi} + h_{ri} + h_{di} \tag{11}$$

We derive the cost function to estimate the sun vector using formula (11):

$$J = \sum_{i=1}^{m}\left(\frac{h_i}{h_0} - \frac{I_i}{I_{i\max}}\right)^2 \tag{12}$$

where m is the number of the light receiving solar panels and h_0 is the irradiance when $\theta_i = 0$. $I_{i\max}$ is the maximum current of each solar panel and I_i is the current of each solar panel obtained using current sensors. This cost function J includes the sun vector $\mathbf{S}_r$, which includes the two variables, the sun altitude θ_s and sun azimuth α_s. Each normal vector of a solar panel is known, $I_{i\max}$ is the maximum current obtained using the two-diode model, and I_i is obtained using a current sensor. Because the cost function J represents the error between the model function and the measurement data, the optimum sun altitude and azimuth are estimated as a pair which minimizes the cost function J. Because the variables included in the cost function are θ_s and α_s, it is possible to solve this problem as a minimization problem of two variables. We use the Nelder-Mead Simplex method [15] to minimize the cost function. The sun vector $\mathbf{S}_r$ is then estimated by substituting θ_s and α_s, obtained using the minimization of cost function J, into formula (2).

In addition, the gravity vector for the rover frame can be obtained using the inclinometer data. The gravity vector in the topocentric frame is along $-Z_t$, and the relationship to the rover frame is represented as follows:

$$\mathbf{G}_r = \mathbf{A}\mathbf{G}_t + \mathbf{v}_g \tag{13}$$

where $\mathbf{G}_r$ is the gravity vector in the rover frame, $\mathbf{G}_t$ is the gravity vector in the topocentric frame, $\mathbf{A}$ is the direction cosine matrix, and $\mathbf{v}_g$ is the sensor noise.

Thus we can estimate the sun vector and gravity vector in the rover frame. With these vectors, the attitude of the rover is estimated using the q method, which uses the vectors in the reference frame and the measured or estimated vectors, and also obtains the direction cosine matrix $\mathbf{A}$ by minimizing the loss function $L(\mathbf{A})$ given by:

$$L(\mathbf{A}) = \frac{1}{2}\sum_{i=1}^{n} a_i \left|\hat{\mathbf{W}}_i - \mathbf{A}\hat{\mathbf{V}}_i\right|^2 \tag{14}$$

where a_i are the corresponding weights, n is the number of vectors (sun vector and gravity vector, so $n = 2$), $\hat{\mathbf{W}}_i$ are the observation unit vectors which are measured in the rover frame, $\hat{\mathbf{V}}_i$ are the reference unit vectors in the reference frame in which we want to estimate attitude, which is the topocentric frame in Fig. 1, and the sun vector in an arbitrary position and time can be obtained from ephemeris [16]. The direction cosine matrix in formula (14) is described by the quaternion $\tilde{\mathbf{q}}$. Here the quaternion is represented as:

$$\tilde{\mathbf{q}} = \left[\tilde{q}_1 \quad \tilde{q}_2 \quad \tilde{q}_3 \quad \tilde{q}_4\right]^T \tag{15}$$

$$\mathbf{Q} = \left[\tilde{q}_1 \quad \tilde{q}_2 \quad \tilde{q}_3\right]^T \tag{16}$$

$$\tilde{\mathbf{q}}^T\tilde{\mathbf{q}} = 1 \tag{17}$$

The quaternion has a constraint represented by formula (17). This means that the loss function in formula (14) can be represented as a constrained optimization problem. Here rewriting formula (14) using the Lagrange multiplier method derives an equation given by:

$$J(\tilde{\mathbf{q}}) = \tilde{\mathbf{q}}^T\mathbf{K}\tilde{\mathbf{q}} - \lambda\left(\tilde{\mathbf{q}}^T\tilde{\mathbf{q}} - 1\right) \tag{18}$$

where λ is the Lagrange multiplier and $\mathbf{K}$ is a 4×4 symmetric matrix given by:

$$\mathbf{K} = \begin{bmatrix} \mathbf{S}-\sigma\mathbf{I} & \mathbf{Z} \\ \mathbf{Z}^T & \sigma \end{bmatrix} \tag{19}$$

with

$$\sigma = \mathrm{trace}(\mathbf{B}) \tag{20}$$

$$\mathbf{B} = \sum_{i=1}^{2} a_i \hat{\mathbf{W}}_i \hat{\mathbf{V}}_i \tag{21}$$

$$a_1 = \frac{\sigma_1^{-2}}{\sum_{i=1}^{2} \sigma_i^{-2}} \tag{22}$$

$$a_2 = \frac{\sigma_2^{-2}}{\sum_{i=1}^{2} \sigma_i^{-2}} \tag{23}$$

$$\mathbf{S} = \mathbf{B}^T + \mathbf{B} \tag{24}$$

$$\mathbf{Z} = \left[B_{2,3} - B_{3,2} \quad B_{3,1} - B_{1,3} \quad B_{1,2} - B_{2,1} \right]^T \tag{25}$$

where σ_1^2 and σ_2^2 are the observation-error variance of the sun vector and the gravity vector respectively.

Differentiating formula (18) shows that $J(\tilde{\mathbf{q}})$ has a solution when:

$$\mathbf{K}\tilde{\mathbf{q}} = \lambda \tilde{\mathbf{q}} \tag{26}$$

Thus $\tilde{\mathbf{q}}$ denotes an eigenvector of $\mathbf{K}$. The optimal quaternion $\tilde{\mathbf{q}}_{opt}$ is chosen to be the eigenvector of $\mathbf{K}$ belonging to its largest eigenvalue.

The relationship between $\tilde{\mathbf{q}}$ and direction cosine matrix $\mathbf{A}(\tilde{\mathbf{q}})$ described by the quaternion is given by:

$$\mathbf{A}(\tilde{\mathbf{q}}) = \left(\tilde{q}_4^{\,2} - \mathbf{Q} \cdot \mathbf{Q} \right) \mathbf{I} + 2\mathbf{Q}\mathbf{Q}^T + 2\tilde{q}_4 [\tilde{\mathbf{q}} \times] \tag{27}$$

where $[\tilde{\mathbf{q}} \times]$ is an antisymmetric matrix given by:

$$[\tilde{\mathbf{q}} \times] = \begin{bmatrix} 0 & \tilde{q}_3 & -\tilde{q}_2 \\ -\tilde{q}_3 & 0 & \tilde{q}_1 \\ \tilde{q}_2 & -\tilde{q}_1 & 0 \end{bmatrix} \tag{28}$$

Using the shorthand $\mathrm{c}\,\theta = \cos\theta$, and $\mathrm{s}\,\theta = \sin\theta$, $\mathbf{A}(\tilde{\mathbf{q}})$ can be converted into Euler angle formulation given by:

$$\mathbf{A}(\tilde{\mathbf{q}}) = \begin{bmatrix} \mathrm{c}\theta_r\,\mathrm{c}\psi_r & \mathrm{c}\theta_r\,\mathrm{s}\psi_r & -\mathrm{s}\theta_r \\ \mathrm{s}\phi_r\,\mathrm{s}\theta_r\,\mathrm{c}\psi_r - \mathrm{c}\phi_r\,\mathrm{s}\psi_r & \mathrm{s}\phi_r\,\mathrm{s}\theta_r\,\mathrm{s}\psi_r + \mathrm{c}\phi_r\,\mathrm{c}\psi_r & \mathrm{s}\phi_r\,\mathrm{c}\theta_r \\ \mathrm{c}\phi_r\,\mathrm{s}\theta_r\,\mathrm{c}\psi_r + \mathrm{s}\phi_r\,\mathrm{s}\psi_r & \mathrm{c}\phi_r\,\mathrm{s}\theta_r\,\mathrm{s}\psi_r - \mathrm{s}\phi_r\,\mathrm{c}\psi_r & \mathrm{c}\phi_r\,\mathrm{c}\theta_r \end{bmatrix} \tag{29}$$

where ϕ_r, θ_r, and ψ_r denote the roll, pitch and yaw angles respectively, described as:

$$\phi_r = \tan^{-1}\left(\frac{A_{2,3}}{A_{3,3}} \right) \tag{30}$$

$$\theta_r = -\sin^{-1} A_{1,3} \tag{31}$$

$$\psi_r = \tan^{-1}\left(\frac{A_{1,2}}{A_{1,1}} \right) \tag{32}$$

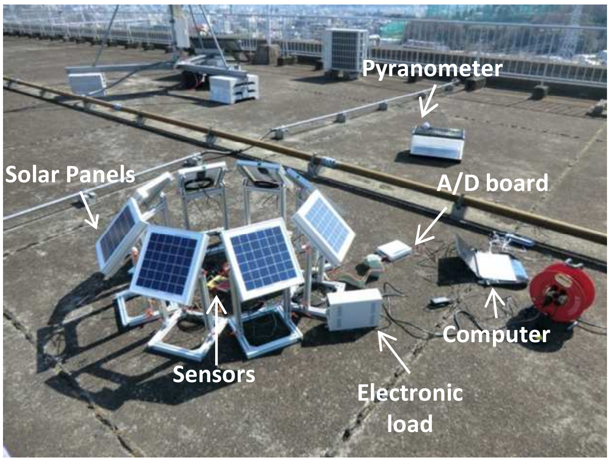

Figure 5. Experiment platform (shown here on Keio University)

4 Attitude determination platform

To validate the attitude determination system, some experiments with solar panels and sensors were carried out. In this chapter, we describe the experimental setup and our means of determining the true values of the attitude, and experiment conditions.

4.1 Experiment system

The experiments were conducted at Keio University (35° 33.3′ N latitude and 139° 39.2′ E longitude). The experiments data were collected by experiment platform shown in Fig. 5. The platform includes solar panels, a pyranometer, an accelerometer as an inclinometer, an electronic load, and a laptop computer to collect these data. Although this system was not an actuated rover, our focus with this platform is on problems of attitude determination, and thus it was sufficient as a means to collect data.

We measured the true north before the experiments were conducted to obtain the yaw true value of the attitude. Because magnetometer contains magnetic declination, we measured the direction of a shadow of a vertical rod at the time of culmination and we considered this direction as the true north. Therefore the yaw true value is 0 degree. The true values of roll and pitch angle were the average values of 18000 samples which obtained by an inclinometer.

4.2 Experiment conditions

To validate the effectiveness of the proposed attitude determination system, the experiments were conducted in the six conditions shown in Table 1. In each condition, voltage, current, and temperature of each solar panel, irradiance, and acceleration were logged at every 1 second.

Table 1. Experiment condition of solar panels

Experiment No.	Number of solar panels	Angle of solar panels [deg]
1	4	45
2	8	45
3	4	60
4	8	60
5	4	30
6	8	30

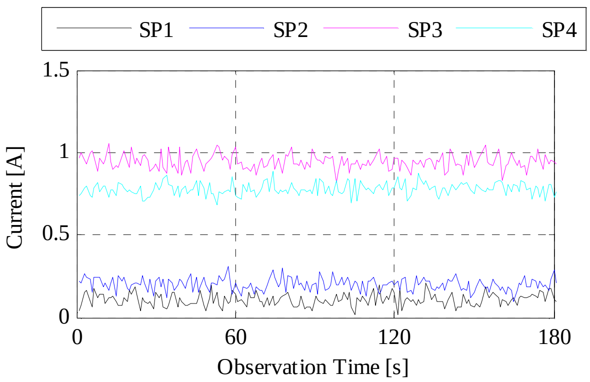

Figure 6. Output current of solar panels (Experiment 1, SP: Solar Panel)

Table 2. Current error between model and measured value

Current Error	Mean [A]	Standard Deviation [A]
Solar Panel 1	0.083	0.038
Solar Panel 2	0.046	0.038
Solar Panel 3	0.024	0.045
Solar Panel 4	-0.013	0.038

5 Experiment results

This section presents experimental results using the proposed attitude determination system. Fig. 6 shows current of each solar panel (SP) for Experiment 1. It was confirmed from Fig. 6 that the current of each solar panel is different depending on the angles of incidence for each solar panel. Table 2 shows the mean values and standard deviations of the current errors between the model described in chapter 2 and measured values.

We applied the proposed system to the current data and estimated the attitude. Fig. 7 shows the experiment results for Experiment 1. Although the estimate values for the roll and pitch angle have small errors due to high precision of the inclinometer to obtain the gravity vector, the estimate values of the yaw angle include relatively large noise. This is because the current sensors have noise shown in Fig. 6. In this study, because we assumed the rovers do not move during attitude determination, the most appropriate attitude estimate values at time *t* were obtained using a filter which derives the mean values of all estimate values obtained before time *t*. Fig. 8 shows the results using this filter. The estimate values for the roll, pitch and yaw angle converge

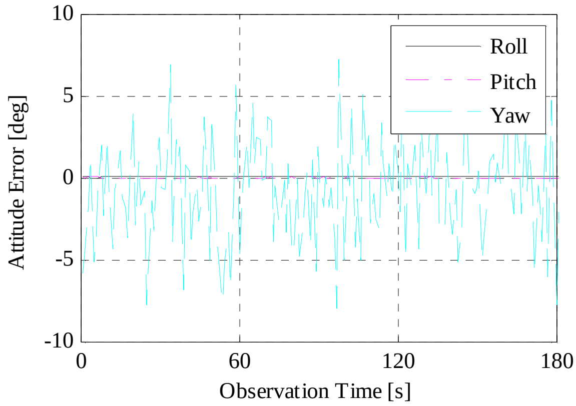

Figure 7. Time history of the attitude estimate error (Experiment 1, raw)

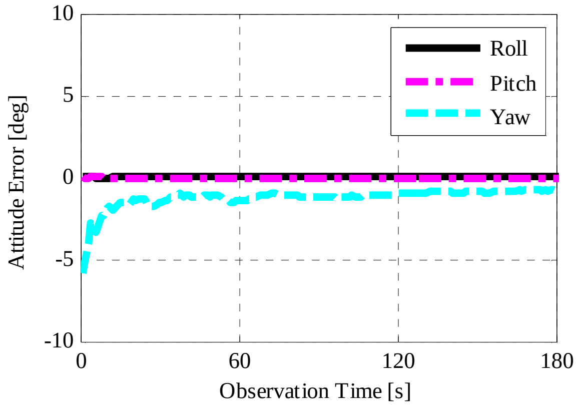

Figure 8. Time history of the attitude estimate error (Experiment 1, filtered)

Table 3. Attitude estimate error at 180 s

Experiment No.	Roll Error [deg]	Pitch Error [deg]	Yaw Error [deg]
1	0.07	-0.03	-0.82
2	0.02	0.01	-1.21
3	-0.03	0.07	-4.97
4	-0.03	0.06	-1.89
5	-0.03	0.04	-4.56
6	0.02	0.02	-3.06

in approximately 60 s.

The estimate errors for all experiments are shown in Table 3. It was confirmed from Table 3 that the estimate errors for the yaw angle are larger than the roll and pitch angle. Although the estimate errors are small for Experiments 1, 2 and 4, they deteriorate for Experiment 3. This is because only two solar panels received sunlight. In Experiments 5 and 6, the estimate errors are relatively large. The cause is as follows: the panels are nearly horizontal in Experiment 5 and 6, and current of solar panels is barely depended on the direction of the sun.

6 Conclusion

We have proposed an attitude determination system as a redundant system that employs the characteristics of solar panels and an inclinometer in planetary exploration rovers. In our proposed system, the solar panels' current, voltage, temperature and normal vectors were employed to estimate the sun vector, and an inclinometer was used to obtain the gravity vector. The attitude of a rover can be estimated using the q method with the two vectors. Because this attitude determination system does not need any external sensors such as cameras or sun sensors, but only internal sensors, it is easy to apply to rovers. Because we use the solar panels' normal vectors, the attitude determination system can be applied to various shapes of rovers. The experiments results indicate that the proposed attitude determination system can estimate the roll, pitch, and yaw angles within a few degrees of accuracy. It follows from this that adequate accuracy can be achieved with the attitude determination system which uses solar panels.

Because this system is assumed that the position of the rover is known, it is essential that this system is combined with the localization system in order to build the integrated navigation system.

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