APPROACH METHOD OF A FORMATION SPACE ROBOTIC SYSTEM FOR ON-ORBIT SERVICING OF GEO

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ABSTRACT

In this paper, we present a concept of formation space robotic system for on-orbit servicing (OOS) of non-cooperative target in geostationary orbit (GEO). The formation space robotic system composes of an operation spacecraft and a monitor space robot. Given the relative motion among monitor spacecraft, operation space robot and non-cooperative target, an approach model of triangle formation is derived. Then, a novel formation approach scheme of relative navigation and guidance is proposed. It contains a distributed relative navigation algorithm based on multiple line-of-sights (LOS) and a multiple velocity impulses relative guidance law based on Particle Swarm Optimization (PSO). The semi-physical simulation results show that the far range rendezvous between formation space robotic system and non-cooperative target can be realized autonomously by the proposed formation approach scheme. The formation space robotic system can improve the observability of angles-only relative navigation and the precision of two velocity impulses relative guidance to expand the ability of autonomous rendezvous.

1. INTRODUCTION

According to the report by European Space Agency (ESA) [1], there were 1369 objects in GEO of which only 31% were controlled operational satellites. As part of spent objects, the malfunctioned satellites may fail to correctly deploy stowed appendages (such as solar arrays or antennas) at the beginning of life (BOL), lose ability to de-orbit at the ending of life (EOL), suffer degraded capabilities due to space environment, etc. Therefore, more and more attentions have been paid to OOS of GEO. Space robotic systems are expected to play an increasingly important role in OOS [2]-[3].

To service a target, a space robotic system should have the ability of relative navigation and guidance. However, most spacecrafts of GEO are non-cooperative target. In far range rendezvous, only LOS measurement devices can be used for relative navigation. Lacking of range information, it is difficult for a space robotic system to perform relative navigation by single LOS in GEO. The weak observability of single LOS (Angles-only) relative navigation was analysed in Ref. [4]-[5]. Some orbital maneuver methods were given to improve observability. A multiple LOS relative navigation method by formation satellites was proposed in Ref. [6]-[7]. The information of baseline between formation satellites was introduced into measurement equation. A centralized EKF which had 12 state variables was established to realize relative navigation. Meanwhile, the relative guidance between space robotic system and target becomes particularly important. In low earth orbit (LEO), the relative guidance law is usually C-W equation. The optimal solutions of two velocity impulses and time-fixed were widely developed. e. g. Genetic Algorithms (GA) were studied to solve the optimal solution of two velocity impulses. In GEO, the constraint of C-W equation is weaker than that in LEO. The farther the range of rendezvous is, the lower the precision of two velocity impulses C-W guidance is. A space robotic system cannot reach the desired position by two velocity impulses C-W guidance in GEO. Several supplementary velocity impulses should be added into C-W guidance to modify final position [8].

Therefore, the task space of single space robotic system is limited in GEO. To improve the autonomy of OOS system, this paper presents conceptual design of a formation space robotic system that can prolong the range of autonomous rendezvous for non-cooperative targets. The formation space robotic system which composes of a monitor spacecraft and an operation space robot is described in section 2. For relative navigation, a distributed EKF algorithm based on multiple LOS is proposed in section 3. A multiple velocity impulses C-W guidance law based on PSO is studied in section 4. In Section 5, a semi-physical simulation system is designed to verify the proposed approach scheme in a far range rendezvous mission of GEO. The paper is concluded in section 6, with an overall reflection on the framework for the development of formation space robotic system.

2. CONCEPTUAL DESIGN OF FORMATION SPACE ROBOTIC SYSTEM

2.1. The configuration of formation space robotic system

The formation space robotic system composes of a monitor spacecraft and an operation space robot that are connected by inter-satellite link. For far range rendezvous, the triangle configuration between the system and a non-cooperative target is shown in Figure 1. Learning from the experience of above-mentioned OOS projects of GEO, the definitions of OOS missions for non-cooperative targets are shown in Table 1. The OOS missions include Rendezvous & approach, Fly-around & Fly-by, Docking & capturing and Servicing & retreating.

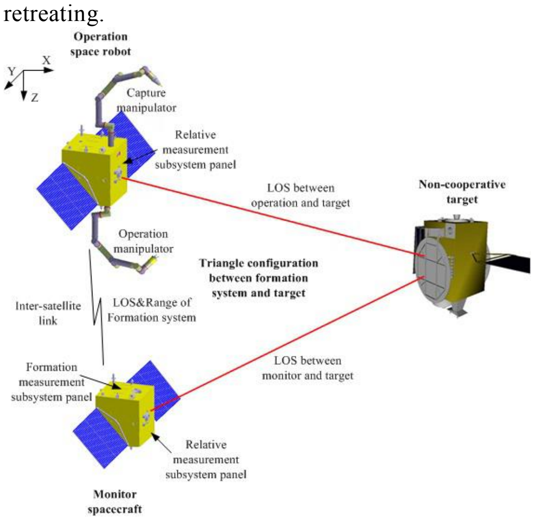

Figure 1: Triangle configuration between formation space robotic system and target.

Table 1: The definition of OOS mission

Name	Content	Implementation
Rendezvous & approach	Far range rendezvous from 200km to 20km	Formation space robotic system
Rendezvous & approach	Medium range rendezvous from 20km to 40m	Operation space robot and monitor spacecraft separately
Fly-around & Fly-by	Flying around target on a circle with radius of 40m, Flying by target at 40m	Monitor spacecraft
Rendezvous & approach	Close range rendezvous from 40m to 2m	Operation space robot
Docking & capturing	Docking from 2m to 0.5m, capturing with manipulator	Operation space robot
Servicing & retreating	Inspecting and Servicing with manipulator, retreating after servicing	Formation space robotic system

The functions of operation space robot with two manipulators are rendezvous, capturing and servicing. The operation space robot includes two 7-DOF manipulators, relative measurement subsystem, GNC (Guidance, Navigation and Control) subsystem and platform subsystem. The functions of monitor spacecraft without manipulator are rendezvous, fly-around monitoring and fly-by observing. The operation space robot includes relative measurement subsystem, formation measurement subsystem, GNC (Guidance, Navigation and Control) subsystem and platform subsystem. Far range autonomous rendezvous without ground intervention is the main character of the formation system. The relative navigation and guidance methods are key techniques in far range autonomous rendezvous. Before proposing these methods, the relative dynamics between formation space robotic system and target is introduced firstly.

2.2. The relative dynamics between formation system and target

1) The model of triangle formation

The in-plane relative motion of triangle configuration between formation space robotic system and target is portrayed in Figure 2. The relative coordinate system of spacecraft is $\sum i$-XYZ. The origin $\sum i$ is the central body of spacecraft. The X-axis points along the velocity vector of spacecraft. The Z-axis is towards the central body of the Earth. And the Y-axis completes the right handed system. L_i is the LOS among the three spacecrafts. $\boldsymbol{r}_i$ is the position vector from the central body of spacecraft to the Earth. The azimuth angles (α_1, β_1) and (α_2, β_2) between formation space robotic system and target can be obtained by the relative measurement subsystems. The azimuth angle (α_3, β_3) and relative distance ρ_3 of formation space robotic system can be obtained by the formation measurement subsystem of monitor spacecraft. The relative distance ρ_1 and ρ_2 between formation space robotic system and target are unknown that will be obtained by the multiple LOS relative navigation method.

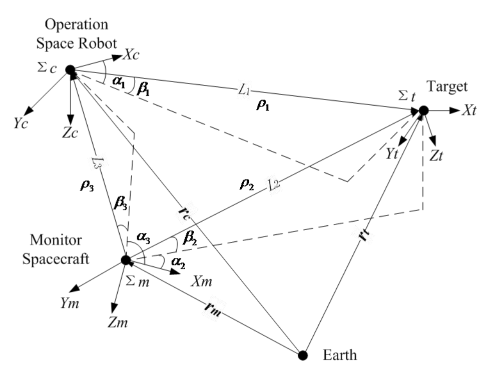

Figure 2: Relative motion between formation system and target

2) The relative dynamics

In a circular orbit, the linear relative dynamics of a chase spacecraft with respect to a target spacecraft can be described by the C-W equations when the relative distance is far less than the orbit radius. The orbit radius of GEO spacecraft is about 40,000 kilometers. And the far range rendezvous of the formation space robotic system is about 200 kilometers. Therefore, the C-W equations can be used to describe the relative dynamics. Taking the C-W equations between operation space robot and target as an example:

$$\begin{cases} \ddot{x} = 2n\dot{z} + a_x \\ \ddot{y} = -n^2 y + a_y \\ \ddot{z} = 3n^2 z - 2n\dot{x} + a_z \end{cases}$$

* MERGEFORMAT (1)

In equation (1), $[x,y,z]^{\mathrm{T}}$ is relative position. n is the orbital angular rate of target. $[a_x,a_y,a_z]^{\mathrm{T}}$ are external accelerations except the earth's gravitational acceleration.

It is assumed that equation is the system equation. The velocities can be computed by taking the first time derivative of position. Then, the state vector can be defined as:

$$\boldsymbol{X} = [\boldsymbol{r},\boldsymbol{v}]^{\mathrm{T}} = [x,y,z,\dot{x},\dot{y},\dot{z}]^{\mathrm{T}}$$

* MERGEFORMAT (2)

If there are no external accelerations, the solution of system equation is:

$$\boldsymbol{X}(t) = \boldsymbol{\Phi}(t)\boldsymbol{X}(0)$$

* MERGEFORMAT (3)

In equation , $\boldsymbol{\Phi}(t)$ is the state transition matrix.

$$\boldsymbol{\Phi}(t) = \begin{bmatrix} 1 & 0 & 6\tau - 6\boldsymbol{S}\tau & \frac{4\boldsymbol{S}\tau - 3\tau}{n} & 0 & \frac{2-2\boldsymbol{C}\tau}{n} \\ 0 & \boldsymbol{C}\tau & 0 & 0 & \frac{\boldsymbol{S}\tau}{n} & 0 \\ 0 & 0 & 4-3\boldsymbol{C}\tau & \frac{-2+2\boldsymbol{C}\tau}{n} & 0 & \frac{\boldsymbol{S}\tau}{n} \\ 0 & 0 & 6n - 6n\boldsymbol{C}\tau & 4\boldsymbol{C}\tau - 3 & 0 & 2\boldsymbol{S}\tau \\ 0 & -n\boldsymbol{S}\tau & 0 & 0 & \boldsymbol{C}\tau & 0 \\ 0 & 0 & 3n\boldsymbol{S}\tau & -2\boldsymbol{S}\tau & 0 & \boldsymbol{C}\tau \end{bmatrix}$$

* MERGEFORMAT (4)

Where, $\boldsymbol{S}$ stands for sine, $\boldsymbol{C}$ stands for cosine, $\tau = nt$, $\boldsymbol{\Phi}(t)$ is a time varying matrix. In the following parts, the relative navigation and guidance methods based on C-W equations will be presented.

3. RELATIVE NAVIGATION BASED ON MULTIPLE LOS

The weak observability of single LOS relative navigation was analyzed in Ref.[4]-[5]. Therefore, using single LOS, it is difficult for a space robot to perform autonomous relative navigation of non-cooperative target in GEO. The formation space robotic system can provide two LOS of target and a baseline to overcome the problem reasonably. Meanwhile, to reduce the state dimension and complexity of centralized filter, distributed EKFs that separately work in operation space robot and monitor spacecraft are proposed.

3.1 The measurement equations of formation space robotic system

1) The measurement equation of formation navigation

For formation flying, the relative position and velocity between operation space robot and monitor spacecraft should be obtained. The EKF method based on Angles-Range is usually used for relative navigation. By the formation measurement subsystem of monitor spacecraft, the azimuth angle (α_3,β_3) and relative distance ρ_3 can be gotten in real time. The state vector is $\boldsymbol{X}_{mc} = [x_{mc},y_{mc},z_{mc},\dot{x}_{mc},\dot{y}_{mc},\dot{z}_{mc}]^{\mathrm{T}}$. The models of LOS and range are given as:

$$\begin{cases} \alpha_3 = \arctan(-z_{mc} / x_{mc}) + w_{\alpha 3} \\ \beta_3 = \arctan(y_{mc} / \sqrt{x_{mc}^2 + z_{mc}^2}) + w_{\beta 3} \\ \rho_3 = \sqrt{x_{mc}^2 + y_{mc}^2 + z_{mc}^2} + w_{\rho 3} \end{cases}$$

* MERGEFORMAT (5)

Then, the measurement equation can be rewritten as:

$$\boldsymbol{Z}_{mc} = \boldsymbol{h}(x_{mc},y_{mc},z_{mc}) + \boldsymbol{w}_{mc}$$

* MERGEFORMAT (6)

In equation , $\boldsymbol{Z}_{mc} = [\alpha_3,\beta_3,\rho_3]^{\mathrm{T}}$ is measurement vector. $\boldsymbol{w}_{mc} = [w_{\alpha 3},w_{\beta 3},w_{\rho 3}]^{\mathrm{T}}$ is measurement noise vector that is White Gaussian noise. $\boldsymbol{h}(\bullet)$ is a nonlinear measurement equation. For EKF, the nonlinear measurement equation can be linearized by Jacobi local linearization method.

2) The measurement equation of relative navigation

For far range rendezvous, the relative position and velocity between formation space robotic system and target must be obtained. By the relative measurement subsystem of operation space robot, the azimuth angle (α_1,β_1) can be measured. However, it is difficult for operation space robot to compute relative position and velocity of target by using the single LOS. To solve the problem, the azimuth angle (α_2,β_2) measured by monitor spacecraft will be added to assist in the relative navigation of target.

By equation and , the inner relative state $\boldsymbol{X}_{mc}$ of formation space robotic system can be filtered firstly. Then, the relationship between monitor spacecraft and target is:

$$\begin{bmatrix} x_{mt} \\ y_{mt} \\ z_{mt} \end{bmatrix} = \boldsymbol{M}_{mc} \begin{bmatrix} x_{ct} \\ y_{ct} \\ z_{ct} \end{bmatrix} + \begin{bmatrix} x_{mc} \\ y_{mc} \\ z_{mc} \end{bmatrix}$$
* MERGEFORMAT (7)

In equation , $\boldsymbol{M}_{mc} = \boldsymbol{M}_{mi}\boldsymbol{M}_{ic}$. $\boldsymbol{M}_{mi}$ is the attitude rotation matrix of monitor spacecraft in inertia coordinate system. $\boldsymbol{M}_{ic}$ is the attitude rotation matrix of operation space robot in inertia coordinate system. $\boldsymbol{X}_{ct} = [x_{ct}, y_{ct}, z_{ct}, \dot{x}_{ct}, \dot{y}_{ct}, \dot{z}_{ct}]^{\mathrm{T}}$ is the state vector between operation space robot and target. $\boldsymbol{X}_{mt}$ is the state vector between monitor spacecraft and target. Then, the integrated measurement models are:

$$\begin{cases} \alpha_1 = \arctan(-z_{ct} / x_{ct}) + w_{\alpha 1} \\ \beta_1 = \arctan(y_{ct} / \sqrt{x_{ct}^2 + z_{ct}^2}) + w_{\beta 1} \\ \alpha_2 = \arctan(-z_{mt} / x_{mt}) + w_{\alpha 2} \\ \beta_2 = \arctan(y_{mt} / \sqrt{x_{mt}^2 + z_{mt}^2}) + w_{\beta 2} \end{cases}$$
* MERGEFORMAT (8)

Substituting equation to , the measurement equation of state vector $\boldsymbol{X}_{ct}$ is:

$$\boldsymbol{Z}_{ct} = \boldsymbol{h}(\boldsymbol{X}_{ct}, \boldsymbol{M}_{mc}, \boldsymbol{X}_{mc}) + \boldsymbol{w}_{ct}$$
* MERGEFORMAT (9)

In equation , $\boldsymbol{Z}_{ct} = [\alpha_1, \beta_1, \alpha_2, \beta_2]^{\mathrm{T}}$ is measurement vector. $\boldsymbol{w}_{ct} = [w_{\alpha 1}, w_{\beta 1}, w_{\alpha 2}, w_{\beta 2}]^{\mathrm{T}}$ is also measurement noise vector that is White Gaussian noise. For EKF, the matrix $\boldsymbol{M}_{mc}$ and the state $\boldsymbol{X}_{mc}$ are as known parameters.

The state dimension of formation and relative navigation are only 6. The complexity of EKF can be reduced obviously to realize a distributed EKF algorithm.

3.2 The distributed EKF algorithm of navigation

Two EKFs are separately distributed into the formation space robotic system. The EKF of formation navigation is in monitor spacecraft. The EKF of relative navigation for target is in operation space robot. The process of distributed EKF algorithm is described as follows:

Step 1: the monitor spacecraft measures the azimuth angle (α_3, β_3) and relative distance ρ_3 of operation space robot, the azimuth angle (α_2, β_2) of target. It also computes the attitude rotation matrix $\boldsymbol{M}_{mi}$ in inertia coordinate system.

Step 2: the monitor spacecraft uses EKF to filter the relative state $\boldsymbol{X}_{mc}$ of formation space robotic system. It transfers the information of $\boldsymbol{X}_{mc}$, (α_2, β_2) and $\boldsymbol{M}_{mi}$ to operation space robot by inter-satellite link.

Step 3: Thirdly, the operation space robot measures the azimuth angle (α_1, β_1) of target. It also computes the attitude rotation matrix $\boldsymbol{M}_{ic}$ in inertia coordinate system.

Step 4: Fourthly, by receiving the information from monitor spacecraft, the operation space robot computes the transform matrix $\boldsymbol{M}_{mc}$ and substitutes (α_1, β_1) , $\boldsymbol{M}_{mc}$ and (α_2, β_2) into EKF to filter the relative state $\boldsymbol{X}_{ct}$ of target.

In next section, a multiple velocity impulses C-W relative guidance law is proposed to satisfy the triangle formation configuration.

4. MULTIPLE IMPULSES GUIDANCE BASED ON PSO

To improve the guidance precision of far range autonomous approach to non-cooperative target under multiple LOS navigation, a multiple velocity impulses C-W guidance law with mid-correction is proposed. The multiple velocity impulses question of fixed-time fuel optimal is presented firstly. And a PSO algorithm that has two fitness functions is introduced to solve the multi-objective optimization problem with constraints of guidance precision and angle between double LOS.

3

4

4.1 The multiple velocity impulse guidance

The constraint of C-W equations is weak in GEO. The two velocity impulses C-W guidance law can be disturbed easily by external environment. If velocity impulses are added in the process of rendezvous, the precision of final state will be improved. It is assumed that the rendezvous period from initial state $\boldsymbol{X}_0^-$ to final state $\boldsymbol{X}^+(t)$ is *t*, the number of velocity impulses is $N \geq 2$, the time of *i*th (i =0,1,2...N-1) velocity impulse is t_i. From two velocity impulses C-W guidance [], we can have:

$$\boldsymbol{X}^+(t) = \boldsymbol{\Phi}(t)\boldsymbol{X}_0^- + \sum_{i=0}^{N-1} \boldsymbol{\Phi}(t - t_i) \begin{bmatrix} \boldsymbol{0} \\ \Delta \boldsymbol{v}_i \end{bmatrix}$$
* MERGEFORMAT (10)

$\boldsymbol{X}_0^- = [\boldsymbol{r}_0, \boldsymbol{v}_0^-]^{\mathrm{T}}$ is the initial state. $\boldsymbol{v}_0^-$ is the velocity before impulse control. $\boldsymbol{X}^+(t) = [\boldsymbol{r}_t, \boldsymbol{v}_t^+]^{\mathrm{T}}$ is the desired final state. $\boldsymbol{v}_t^+$ is the desired velocity at final time. $\Delta \boldsymbol{v}_i$ is velocity impulses.

4.2 The optimization question of multiple impulses

To reduce the complexity of optimization question, the distributed optimization methods will separately work in operation space robot and monitor spacecraft.

1) The optimization question of operation space robot

For operation space robot, if the initial state ${}^{\mathrm{C}}\boldsymbol{X}_0^{-}=[{}^{\mathrm{C}}\boldsymbol{r}_0, {}^{\mathrm{C}}\boldsymbol{v}_0^{-}]^{\mathrm{T}}$, the final state ${}^{\mathrm{C}}\boldsymbol{X}_t^{+}=[{}^{\mathrm{C}}\boldsymbol{r}_t, {}^{\mathrm{C}}\boldsymbol{v}_t^{+}]^{\mathrm{T}}$ and the rendezvous period ${}^{\mathrm{C}}t$ are given, the optimization question can be described to minimize the sum of velocity impulses. The objective function is:

$$ {}^{\mathrm{C}}J = \min(\sum_{i=0}^{N-1} \left|\Delta {}^{\mathrm{C}}\boldsymbol{v}_i\right|) = \min f({}^{\mathrm{C}}\boldsymbol{y}) $$

* MERGEFORMAT (11)

Where, ${}^{\mathrm{C}}\boldsymbol{y} = ({}^{\mathrm{C}}t_0, \Delta {}^{\mathrm{C}}\boldsymbol{v}_0, \cdots, {}^{\mathrm{C}}t_i, \Delta {}^{\mathrm{C}}\boldsymbol{v}_i),\ i=0,1,\cdots N-1 (N>2)$ are the optimized variables.

Because of navigation error and control error, the operation space robot will not arrive at the desired position exactly. The restriction function of final position is introduced:

$$ P({}^{\mathrm{C}}\boldsymbol{y}) = \left|{}^{\mathrm{C}}\boldsymbol{r}_t - {}^{\mathrm{C}}\boldsymbol{r}_f\right| \le {}^{\mathrm{C}}R $$

* MERGEFORMAT (12)

Where, ${}^{\mathrm{C}}\boldsymbol{r}_f$ is the real position at final time, ${}^{\mathrm{C}}\boldsymbol{r}_t$ is the desired position, ${}^{\mathrm{C}}R$ is the allowable position error.

Then, the solution of multiple impulses guidance is transformed to the question of nonlinear optimal solution with restrictions. In order to simplify the problem, it can be transformed to the question of nonlinear optimal solution with no restriction by penalty function. The new objective function will be:

$$ {}^{\mathrm{C}}T({}^{\mathrm{C}}\boldsymbol{y}, {}^{\mathrm{C}}M) = f({}^{\mathrm{C}}\boldsymbol{y}) + {}^{\mathrm{C}}MP({}^{\mathrm{C}}\boldsymbol{y}) $$

* MERGEFORMAT (13)

Where, ${}^{\mathrm{C}}MP({}^{\mathrm{C}}\boldsymbol{y})$ is the penalty function, ${}^{\mathrm{C}}M$ is the penalty factor.

2) The optimization question of monitor spacecraft

It is assumed that the initial state of monitor spacecraft is ${}^{\mathrm{M}}\boldsymbol{X}_0^{-}=[{}^{\mathrm{M}}\boldsymbol{r}_0, {}^{\mathrm{M}}\boldsymbol{v}_0^{-}]^{\mathrm{T}}$, the final state is ${}^{\mathrm{M}}\boldsymbol{X}_t^{+}=[{}^{\mathrm{M}}\boldsymbol{r}_t, {}^{\mathrm{M}}\boldsymbol{v}_t^{+}]^{\mathrm{T}}$, the rendezvous period is ${}^{\mathrm{M}}t$. Similar to operation space robot, the objective function of monitor spacecraft is:

$$ {}^{\mathrm{M}}J = \min(\sum_{i=0}^{N-1} \left|\Delta {}^{\mathrm{M}}\boldsymbol{v}_i\right|) = \min f({}^{\mathrm{M}}\boldsymbol{y}) $$

* MERGEFORMAT (14)

Where, ${}^{\mathrm{M}}\boldsymbol{y} = ({}^{\mathrm{M}}t_0, \Delta {}^{\mathrm{M}}\boldsymbol{v}_0, \cdots, {}^{\mathrm{M}}t_i, \Delta {}^{\mathrm{M}}\boldsymbol{v}_i),\ i=0,1,2\cdots N-1 (N>2)$ are the optimized variables.

The restriction function of final position is also given as:

$$ P_1({}^{\mathrm{M}}\boldsymbol{y}) = \left|{}^{\mathrm{M}}\boldsymbol{r}_t - {}^{\mathrm{M}}\boldsymbol{r}_f\right| \le {}^{\mathrm{M}}R $$

* MERGEFORMAT (15)

Where, ${}^{\mathrm{M}}\boldsymbol{r}_f$ is the real position at final time, ${}^{\mathrm{M}}\boldsymbol{r}_t$ is the desired position, ${}^{\mathrm{M}}R$ is the allowable position error.

It is different from operation space robot that monitor spacecraft should ensure the triangle formation configuration i.e. the angle between LOS L_1 and L_2 should satisfy the relative navigation. Therefore, the second restriction function of monitor spacecraft is:

$$ P_2({}^{\mathrm{M}}\boldsymbol{y}, {}^{\mathrm{C}}\boldsymbol{y}) = \arccos(\boldsymbol{s}_1 \bullet \boldsymbol{s}_2) \ge \theta $$

* MERGEFORMAT (16)

Where, '$\bullet$' is dot product, θ is the angle restriction value, ${}^{\mathrm{C}}\boldsymbol{y}$ is the optimization result of operation space robot that can be transferred to monitor spacecraft by inter-satellite link.

Then, by penalty function, the new objective function with no restriction of monitor spacecraft will be:

$$ {}^{\mathrm{M}}T({}^{\mathrm{M}}\boldsymbol{y}, {}^{\mathrm{M}}M_1, {}^{\mathrm{M}}M_2, {}^{\mathrm{C}}\boldsymbol{y}) = f({}^{\mathrm{M}}\boldsymbol{y}) + {}^{\mathrm{M}}M_1 P_1({}^{\mathrm{M}}\boldsymbol{y}) + {}^{\mathrm{M}}M_2 P_2({}^{\mathrm{M}}\boldsymbol{y}, {}^{\mathrm{C}}\boldsymbol{y}) $$

* MERGEFORMAT (17)

Where, ${}^{\mathrm{M}}M_1$ and ${}^{\mathrm{M}}M_2$ are the penalty factors.

4.3 The PSO algorithm of multiple impulses

A PSO algorithm is proposed to solve the question of nonlinear optimal solution with no restriction. PSO algorithm has the character of rapid convergence and simple to implementation.

It is assumed that the particle of PSO is $\boldsymbol{y}_i$, the number of maximum iteration is $I_{\max}$, the number of particles is n_{p}. The standard process of PSO algorithm is as follows:

Step 1: Initialization**.** Give a particle swarm and its speeds in random.

$$ \begin{cases} \text{Members:}\left(\boldsymbol{y}_1(k), \boldsymbol{y}_2(k), \cdots, \boldsymbol{y}_{n_p}(k)\right) \\ \text{Speeds:}\left(\Delta\boldsymbol{y}_1(k), \Delta\boldsymbol{y}_2(k), \cdots, \Delta\boldsymbol{y}_{n_p}(k)\right) \end{cases}, 1 \le k \le I_{\max} $$

* MERGEFORMAT (18)

Step 2: Fitness function**.** Compute fitness function of each particle. The fitness function of operation space robot is:

$$ {}^{\mathrm{C}}F = \sum_{i=0}^{N-1} \left| \Delta {}^{\mathrm{C}}\boldsymbol{v}_i \right| + {}^{\mathrm{C}}M_1 \left| {}^{\mathrm{C}}\boldsymbol{r}_t - {}^{\mathrm{C}}\boldsymbol{r}_f \right| $$

* MERGEFORMAT (19)

The fitness function of monitor spacecraft is:

$$ {}^{\mathrm{M}}F = \sum_{i=0}^{N-1} \left| \Delta {}^{\mathrm{M}}\boldsymbol{v}_i \right| + {}^{\mathrm{M}}M_1 \left| {}^{\mathrm{M}}\boldsymbol{r}_t - {}^{\mathrm{M}}\boldsymbol{r}_f \right| + {}^{\mathrm{M}}M_2 \mathrm{acos}(\boldsymbol{s}_1 \bullet \boldsymbol{s}_2) $$

* MERGEFORMAT (20)

Step 3: Local optimal fitness and position**.** Calculate local optimal fitness and position of each particle.

Step 4: Global optimal fitness and position**.** Calculate global optimal fitness and position of particle swarm.

Step 5: Update. Update the particle swarm and its speeds by:

$$ \begin{cases} \Delta \boldsymbol{y}_i(k+1) = \boldsymbol{\omega} \Delta \boldsymbol{y}_i(k) + \boldsymbol{c}_1 Rand_1()(\boldsymbol{P}_{i\_best} - \boldsymbol{y}_i(k)) \\ \qquad + \boldsymbol{c}_2 Rand_2()(\boldsymbol{P}_{g\_best} - \boldsymbol{y}_i(k)) \\ \boldsymbol{y}_i(k+1) = \boldsymbol{y}_i(k) + \Delta \boldsymbol{y}_i(k+1) \end{cases} $$

* MERGEFORMAT (21)

In equation , $\boldsymbol{\omega}$ is the inertia weight. The bigger $\boldsymbol{\omega}$ is, the stronger the global searching ability of PSO is. The smaller $\boldsymbol{\omega}$ is, the stronger the local searching ability of PSO is. $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ represent the speeding figures. The reasonable figures can control the speed of particle's flying and the solution will not fall into local optimal. $Rand_1$ and $Rand_2$ represent random function whose random number is from 0 to 1. Meanwhile, the ranges of optimized variables are given as:

$$ \boldsymbol{y}_i(k+1) = \begin{cases} \boldsymbol{y}_i(k), & \text{if } |\boldsymbol{y}_i(k)| > \boldsymbol{Y}_{\max} \\ \boldsymbol{y}_i(k+1), & \text{else} \end{cases} $$

* MERGEFORMAT (22)

Step 6: Go to step 2 until the number of iteration k is equal to I_{max} or the $\boldsymbol{P}_{g\_best}$ satisfies the requirement of precision.

In next section, the method will be verified by a semi-physical simulation system.

5. SEMI-PHYSICAL SIMULATION AND VERIFICATION

5

5.1 A Semi-physical Vision Simulation System

To verify the conceptual design and approach scheme of formation space robotic system for OOS of GEO, several representative simulations of far range rendezvous to non-cooperative target are presented in this section. All simulations are completed by a semi-physical vision simulation system. Two industrial CCD cameras are introduced to the simulation system to simulate the real process of image acquisition.

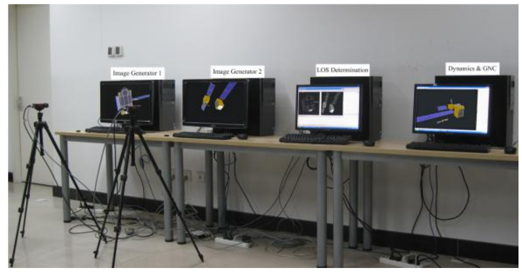

Figure 3: The hardwares of semi-physical vision simulation system

The simulation system is composed of six hardware parts (shown in Figure 3) that are a Dynamics & GNC Computer, two Image Generating & Display Computers, an Image Processing & LOS Determination Computer, and two Industrial CCD Cameras. Their functions are as follows:

1) The Dynamics and GNC Computer

The absolute orbit dynamic of spacecraft is built by ***HPOP*** (High Precision Orbit Prediction) with JGM2 and solar radiation pressure perturbation. Then, relative position vectors among operation space robot, monitor spacecraft and target are:

$$ \begin{cases} \boldsymbol{r}_{ct} = \boldsymbol{r}_c - \boldsymbol{r}_t \\ \boldsymbol{r}_{mt} = \boldsymbol{r}_m - \boldsymbol{r}_t \\ \boldsymbol{r}_{mc} = \boldsymbol{r}_m - \boldsymbol{r}_c \end{cases} $$

* MERGEFORMAT (23)

The functions of GNC are the proposed methods in the paper that are the relative navigation algorithm based on multiple LOS and relative guidance algorithm based on multiple velocity impulses.

2) The Image Generating and Display Computer

Using equation, each computer generates the image of non-cooperative target in background of stars by a virtual camera of OpenGL. The virtual camera simulates a real vision camera mounted on the space robotic system. Then, the image will be projected on a LCD that can be photographed by a CCD camera. The background of stars is generated by **SAO** (The Smithsonian Astrophysical Observatory Star Catalog) that includes the information of 9^{th} magnitude stars.

3) The Industrial CCD Camera

Two real cameras are applied in the simulation system as the physical component in control loop.

The CCD camera 1 stands for relative measurement subsystem of operation space robot. The CCD camera 2 stands for relative measurement subsystem of monitor spacecraft. The process of multiple LOS relative navigation can be reflected more truly. The industrial CCD camera whose type is Manta 146B from AVT (Germany Company) is selected as the real camera.

4) The Image Processing and LOS Determination Computer

The computer receives the gray images from the CCD cameras and does image processing. The step of image processing algorithm includes: Image Segmentation, Morphological Operations and Point Extraction. The results will be sent to the Dynamics & GNC computer to form the closed loop control system.

5.2 The mission design of simulation

The far range rendezvous mission is from 200km to 20km. It is assumed that the initial triangle formation configuration between formation space robotic system and non-cooperative target is established. The period of rendezvous from initial position to final position is about 18000 seconds. The maximum final allowable position error is $R = 2\text{km}$. The minimum angle θ between LOS L_1 and L_2 is about 15°. The precision of formation measurement subsystem is about 0.01°. The precision of carrier phase range finder is about 0.5m. The control error of velocity impulse is $\delta\Delta\boldsymbol{v} = 0.05\Delta\boldsymbol{v}$.

The initial and final relative positions and velocities between operation space robot and target are:

$$\begin{cases} {}^{\mathrm{C}}\boldsymbol{X}_0^- = [200\text{km}, 0\text{km}, -10\text{km}, 1\text{m/s}, 0\text{m/s}, -0.5\text{m/s}]^{\mathrm{T}} \\ {}^{\mathrm{C}}\boldsymbol{X}_t^+ = [20\text{km}, 0\text{km}, 0\text{km}, 0\text{m/s}, 0\text{m/s}, 0\text{m/s}]^{\mathrm{T}} \end{cases}$$

The angle θ is relation to the initial position of monitor spacecraft. Therefore, we transform the angle restriction question to the initial position optimization question. The initial and final relative positions and velocities between monitor spacecraft and target are:

$$\begin{cases} {}^{\mathrm{M}}\boldsymbol{v}_0^- = [1\text{m/s}, 0\text{m/s}, -0.5\text{m/s}]^{\mathrm{T}} \\ {}^{\mathrm{M}}\boldsymbol{X}_t^+ = [0\text{km}, 0\text{km}, -20\text{km}, 0\text{m/s}, 0\text{m/s}, 0\text{m/s}]^{\mathrm{T}} \end{cases}$$

For the distributed relative navigation, the initial covariance matrixes of EKFs are given as:

$$\boldsymbol{P}_{ct} = \text{diag}(20\text{m}, 10\text{m}, 20\text{m}, 0.5\text{m/s}, 0.5\text{m/s}, 0.5\text{m/s})$$
$$\boldsymbol{P}_{mc} = \text{diag}(10\text{m}, 5\text{m}, 10\text{m}, 0.1\text{m/s}, 0.1\text{m/s}, 0.1\text{m/s})$$

5.3 The simulation of relative navigation and guidance

The multiple LOS navigation based on distributed EKF and the three velocity impulses C-W guidance based on PSO will be simulated. According to the PSO algorithm, the optimized variables are:

$$\boldsymbol{y} = (t_0, \Delta\boldsymbol{v}_0, t_1, \Delta\boldsymbol{v}_1, t_2, \Delta\boldsymbol{v}_2)$$

In the optimized variables, t_0=0, t_2=18000, the velocity impulses $\Delta\boldsymbol{v}_1$ and $\Delta\boldsymbol{v}_2$ can be calculated:

$$\begin{cases} \Delta\boldsymbol{v}_1 = \boldsymbol{\Phi}_{rv}^{-1}(t_2 - t_1)[\boldsymbol{r}_t - \boldsymbol{\Phi}_{rr}(t_2 - t_1)\boldsymbol{r}_1] - \boldsymbol{v}_1 \\ \Delta\boldsymbol{v}_2 = \boldsymbol{v}_t^+ - \boldsymbol{\Phi}_{vr}(t_2 - t_1)\boldsymbol{r}_1 - \boldsymbol{\Phi}_{vv}(t_2 - t_1)[\boldsymbol{v}_1 + \Delta\boldsymbol{v}_1] \end{cases}$$

It is assumed that the orbital transfer of rendezvous is in plane. i.e. the velocity impulse of y-axis is zero. Then, the optimized variables of operation space robot can be simplified as:

$${}^{\mathrm{C}}\boldsymbol{y} = {}^{\mathrm{C}}(\Delta{}^{\mathrm{C}}\boldsymbol{v}_0, {}^{\mathrm{C}}t_1) = (\Delta{}^{\mathrm{C}}V_0\cos{}^{\mathrm{C}}\alpha, \Delta{}^{\mathrm{C}}V_0\sin{}^{\mathrm{C}}\alpha, {}^{\mathrm{C}}t_1)$$

The ranges of the optimized variables are:

$$\begin{cases} \Delta{}^{\mathrm{C}}V_0 \in [0, 20]\text{m/s} \\ {}^{\mathrm{C}}\alpha \in [0, 2\pi]\text{rad} \\ {}^{\mathrm{C}}t_1 \in [0, 18000]\text{s} \end{cases}$$

Because of the added initial position optimization question, the optimized variables of monitor spacecraft are:

$$\begin{aligned} {}^{\mathrm{M}}\boldsymbol{y} &= (\Delta{}^{\mathrm{M}}\boldsymbol{v}_0, {}^{\mathrm{M}}t_1, {}^{\mathrm{M}}\boldsymbol{r}_0) \\ &= (\Delta{}^{\mathrm{M}}V_0\cos{}^{\mathrm{M}}\alpha, \Delta{}^{\mathrm{M}}V_0\sin{}^{\mathrm{M}}\alpha, {}^{\mathrm{M}}t_1, {}^{\mathrm{M}}x_0, {}^{\mathrm{M}}z_0) \end{aligned}$$

The ranges of the optimized variables are:

$$\begin{cases} \Delta{}^{\mathrm{M}}V_0 \in [0, 20]\text{m/s} \\ {}^{\mathrm{M}}\alpha \in [0, 2\pi]\text{rad} \\ {}^{\mathrm{M}}t_1 \in [0, 18000]\text{s} \\ {}^{\mathrm{M}}x_0 \in [150, 200]\text{km} \\ {}^{\mathrm{M}}z_0 \in [-60, -40]\text{km} \end{cases}$$

The parameters of PSO are given in Table 2.

Table 2: The parameters of PSO algorithm

Parameters	Operation space robot	Monitor spacecraft
Particles number n_p	60	60
Iteration number I_{max}	300	300
Penalty factor M_1/ M_2	10^5	$10^5/10^5$
Inertia weight $\boldsymbol{\omega}$	0.6	0.6
Speed figure c_1	2	2
Speed figure c_2	2	2

The results of relative navigation based on multiple LOS are shown in Figure 4. The best results of multiple velocity impulses C-W guidance based on PSO are shown in Table 3. The variation of angle between LOS L_1 and L_2 is shown in Figure 5. The rendezvous trajectory of formation space robotic system is shown in Figure 6.

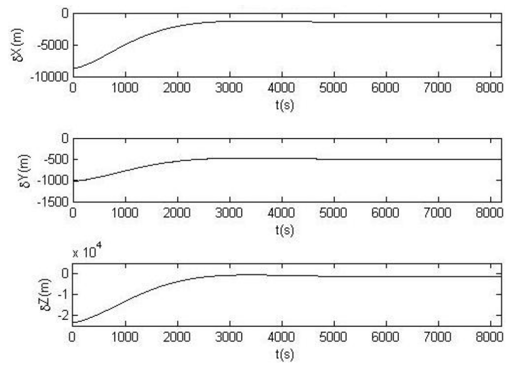

Figure 4: The position errors of multiple LOS

From the results, the relative navigation is convergent. The precision of relative navigation is about 500m. It is concluded that the distributed EFK algorithm of relative navigation based on multiple LOS is effective.

From the results of PSO algorithm, the total velocity increment J of each spacecraft is about 20m/s. The time of additional velocity impulse is close to the end of far range rendezvous that is smaller than the other two. In other words, the rendezvous trajectory is modified to satisfy the precision of final position by the second velocity impulse. The final position of far range rendezvous is less than 2km. The minimum angle between LOS L_1 and L_2 is about 17.2° that satisfies the desired requirement. It is concluded that the multiple velocity impulses C-W relative guidance law based on PSO algorithm is effective.

Table 3. The results of PSO algorithm

Optimized variables	Operation space robot	Monitor spacecraft
t_1(s)	16468	15195
V_0(m/s)	10.786	10.941
a(rad)	3.936	3.417
x_0(km)	/	175.031
z_0(km)	/	-58.953
J(m/s)	22.35	20.475
$\Delta \boldsymbol{v}_0$ (m/s)	$[-7.56,0,-7.70]^T$	$[-10.49,0,-2.96]^T$
$\Delta \boldsymbol{v}_1$ (m/s)	$[0.75,0,-1.94]^T$	$[0.24,-0.35,1.21]^T$
$\Delta \boldsymbol{v}_2$ (m/s)	$[5.22,0,-7.79]^T$	$[3.92,0.29,-7.29]^T$

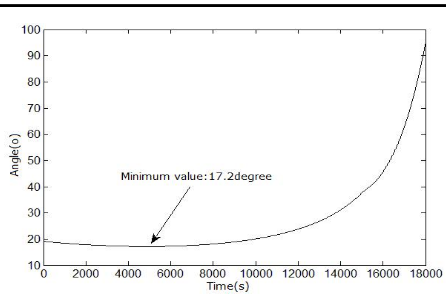

Figure 5: The angle variation between LOS L1 and L2

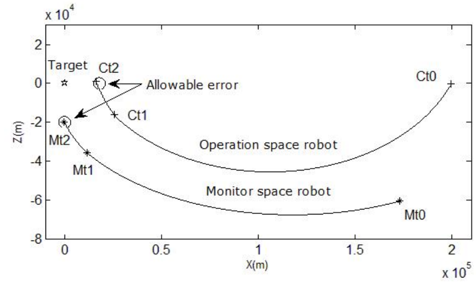

Figure 6: The trajectory of formation space robotic system

6. Conclusion

In this paper, we design a formation space robotic system for OOS of GEO, and develop the key algorithms of relative navigation and guidance for far range autonomous rendezvous.

1) The formation space robotic system can serve most existing satellites of GEO that are with no specially designed objects for measuring and perform completely autonomous rendezvous without ground intervention by triangle formation configuration.

2) The problem of single LOS navigation can be overcome reasonably by distributed relative navigation algorithm based on multiple LOS.

3) The multiple velocity impulses C-W guidance law with mid-correction can improve guidance precision. And a PSO algorithm that has two fitness functions is introduced to solve the multi-objective optimization problem with constraints of guidance precision and angle between double LOS.

4) A semi-physical simulation system based on two visions is designed. By the designed simulation system, the results show that the relative navigation and guidance method of formation space robotic system are effective.

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