BASED ON L-TWO-GAIN ROBUST CONTROLLER FOR FREE-FLOATING MULTIPLE FLEXIBLE-LINKS SPACE ROBOT

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ABSTRACT

The problem of dynamic modelling and trajectory tracking for two flexible links space robot with external bounded disturbance were discussed. The bending vibration of flexible link was mode of simply supported beam. And the dynamical model of the system is successively derived by combining with the system linear momentum, Lagrange equation and assumed mode method. A robust controller which makes the external bounded disturbances have the performance of the L-two-gain of the system was applied to control the system to track the desired trajectory; the closed loop system can restrain the external bounded disturbance signal. Numerical simulation results show that the proposed control algorithm can precision and stability control the joint tracking of the space robot with flexible links, the reliability and validity of the controller are proved.

1 INTRODUCTION

In the future space operation, the space robot is becoming more and more important, and the flexible space robot is the inevitable trend of development. Space robot will play an important role in future space operations, and its application will reduce the risk of astronaut's cabin outer activities and the cost of manned space flight. Because of the special environment of space, there is a strong dynamic coupling between the manipulator and the base. The dynamics and control of space robot system is more complex than the ground. Taking into account the space robot link has the characteristics of light weight, link length, and heavy load and so on, in order to obtain a good control precision and performance of the space robot, it is necessary to consider the flexibility of the robot link. The accuracy and stability of the motion control of space robot will be greatly affected if the flexibility in the links is neglected in the tracking control of the robot system. Therefore, how to establish a dynamic model and design a high performance controller to control the flexible-links is a problem that must be faced and solved in the research and application of space robot [1-5].

As a consequence, structural flexibility in the links is a well known problem, and various control strategies have been proposed [6-8]. The study of control algorithm about space flexible robot mainly concentrated in only with a flexible link, the control of the space robot with two flexible links was also very rare. Su, et al. used the assumed mode method to establish the dynamic modeling of space robot and PD controller was used to control the trajectory tracking control of joint hinge [9]. Yoshisada M, et al. used the adaptive control to achieve vibration suppression of flexible link [10]. Cao, et al. proposed an active vibration control method for flexible manipulator based on fuzzy self-tuning PID algorithm according to the two-links rigid-flexible electric coupling piezoelectricity flexible manipulator [11]. The multiple flexible links space robot system is much more difficult and more challenging than just having single flexible link of space robot system, because of the flexible vibration between the flexible links will inspire each other, the design of corresponding system controller and parameter adjustment is difficult. With the rapid development of space application and space robot, the space robot is more and more used multiple flexible links [12-13].

In order to solve the problem of dynamic modelling and trajectory tracking for two flexible links space robot with external bounded disturbance. A robust controller was designed to achieve the L-two tracking performance with a desired disturbance attenuation level for the flexible-links robot; the closed loop system can restrain the external bounded disturbance signal. Numerical simulation results show that the proposed control algorithm can precision and stability control the joint tracking of the space robot with a two-flexible-link.

2 DYNAMIC MODEL OF FREE-FLOATING FLEXIBLE-LINKS SPACE ROBOT

The maximum number of pages in the manuscript is eight. The structure of free-floating flexible-links space robot of a rigid body satellite B_0 , two flexible links B_1 , B_2 is shown in Fig.1. The links were connected in serial and were actuated by individual actuators. $(O_i - x_i y_i)$ is the principal axis coordinate system of each split body $B_i (i = 1,2)$. $\boldsymbol{x}_i (i = 0,1,2)$ is the symmetrical axis of each link. $\boldsymbol{e}_i$ is the unit vector along the $\boldsymbol{x}_i (i = 0,1,2)$ axis. $\rho_i (i = 1,2)$ is the unit length density of each link. $EI_i (i = 1,2)$ is the flexural rigidity. q_0 is the attitude angle of the base, which is the angle between the X axis and the x_0 axis. $q_i(i=1,2)$ is the i th link angular displacement, i.e. the angle between the x_{i-1} axis and the x_i axis. $\boldsymbol{r}_c$ is the position vector of the mass center C of the entire system in $(O-XY)$ frame.

The flexible link can be regarded as a Euler-Bernoulli beam [14], and based on the assumed mode method the elastic deformation is described as:

$$w_i(x_i,t)=\sum_{j=1}^{n_i}\varphi_{ij}(x_i)\delta_{ij}(t) \tag{1}$$

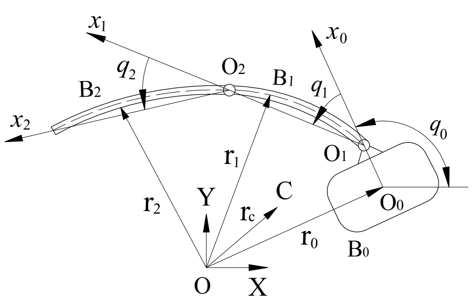

Fig.1 Two flexible links space robot

Where, φ_{ij} is the *j*th mode shape for *i*th link, n_i is the number of mode shapes used to model the lateral deflection of the *i*th link, δ_{ij} is the time varying weight function of φ_{ij}, and $\varphi_{ij}=\sin(j\pi x_i/l)$, $(i=1,2,\cdots,n_i)$. Considering the dominant effect of the low order mode on the elastic vibration of the link, the first two lower order modes are taken into consideration, that is $n_1=n_2=2$.

From position relation of two flexible links space robot, the cancroids of base and any point on the flexible link can be described as

$$\boldsymbol{r}_i=\boldsymbol{r}_c+\sum_{j=0}^{2}L_{ij}\boldsymbol{e}_j \quad (i=0,1,2) \tag{2}$$

Where L_{ij} are the combination inertial parameters of system.

Differentiating Eqs.(2) to time t, velocity vector $\dot{\boldsymbol{r}}_i(i=0,1,2)$ are

$$\dot{\boldsymbol{r}}_i=\boldsymbol{r}_c+\sum_{j=0}^{2}L_{ij}\dot{\boldsymbol{e}}_j \tag{3}$$

According to the knowledge of mechanics, the total kinetic energy of the two flexible link space robot system can be written

$$T=T_0+\sum_{i=1}^{2}T_i \tag{4}$$

Where, $T_0=\frac{1}{2}(m_0\dot{\boldsymbol{r}}_0+j_0\dot{q}_0^2)$， $T_i=\frac{1}{2}\rho_i\int_0^{l_i}\dot{\boldsymbol{r}}_i^2dx_i$ 。

In the space of weak gravity, the gravitational potential energy of the two flexible link space robot system is negligible, and the total potential energy of the system is equal to the bending strain energy of the flexible link.

$$V_\delta=\frac{1}{2}\sum_{i=1}^{2}\left(EI_i\int_0^{l_i}\left(\frac{\partial^2 w_i(x_i,t)}{\partial^2 x_i}\right)^2dx_i\right) \tag{5}$$

The dynamical equation with planar motion of the free-floating space robot with multiple flexible links is successively derived by combining with the system linear momentum, Lagrange equation, and the assumed mode technique. The Euler-Lagrange dynamic equations of the two flexible links space robot are

$$\boldsymbol{M}(\boldsymbol{\theta}_{r\delta})\begin{bmatrix}\ddot{\boldsymbol{\theta}}_r\\ \ddot{\boldsymbol{\theta}}_{\delta1}\\ \ddot{\boldsymbol{\theta}}_{\delta2}\end{bmatrix}+\boldsymbol{H}(\boldsymbol{\theta}_{r\delta},\dot{\boldsymbol{\theta}}_{r\delta})\begin{bmatrix}\dot{\boldsymbol{\theta}}_r\\ \dot{\boldsymbol{\theta}}_{\delta1}\\ \dot{\boldsymbol{\theta}}_{\delta2}\end{bmatrix}+\begin{bmatrix}\boldsymbol{0}_{3\times1}\\ \boldsymbol{K}_{\delta1}\boldsymbol{\theta}_{\delta1}\\ \boldsymbol{K}_{\delta1}\boldsymbol{\theta}_{\delta1}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\tau}\\ \boldsymbol{0}_{4\times1}\end{bmatrix} \tag{6}$$

Where, $\boldsymbol{\theta}_{\delta1}=[\delta_{11}\quad\delta_{12}]^T$ and $\boldsymbol{\theta}_{\delta2}=[\delta_{21}\quad\delta_{22}]^T$ are the first two lower order modes column vector of flexible link B_1 and B_2, respectively; $\boldsymbol{\theta}_r=[q_0\quad q_1\quad q_2]^T$, $\boldsymbol{\theta}_\delta=[\boldsymbol{\theta}_{\delta1}^T\quad\boldsymbol{\theta}_{\delta2}^T]^T$, $\boldsymbol{\theta}_{r\delta}=[\boldsymbol{\theta}_r^T\quad\boldsymbol{\theta}_\delta^T]^T$; $\boldsymbol{M}(\boldsymbol{\theta}_{r\delta})\in R^{7\times7}$ is the links inertia positive-definite matrix; $\boldsymbol{H}(\boldsymbol{\theta}_{r\delta},\dot{\boldsymbol{\theta}}_{r\delta})\in R^{7\times7}$ is the Coriolis/centrifugal matrix; $\boldsymbol{\tau}=[\tau_0\quad\tau_1\quad\tau_2]^T$ is the joint torque/force delivered by actuators; $\boldsymbol{\tau}=[\tau_0\quad\tau_1\quad\tau_2]^T$ is the joint torque/force delivered by actuators; $\boldsymbol{K}_{\delta1}=\mathrm{diag}[k_{11}\quad k_{12}]$ 、 $\boldsymbol{K}_{\delta2}=\mathrm{diag}[k_{21}\quad k_{22}]$ are the stiffness coefficient matrix of flexible link B_1 and B_2, respectively, $\boldsymbol{K}_\delta=\mathrm{diag}[\boldsymbol{K}_{\delta1}\quad\boldsymbol{K}_{\delta2}]$.

3 DESIGN CONTROLLER

According to Eqs. (6), the dynamic equation of the system can be expanded to

$$\begin{bmatrix}\boldsymbol{M}_{11}&\boldsymbol{M}_{12}\\ \boldsymbol{M}_{21}&\boldsymbol{M}_{22}\end{bmatrix}\begin{bmatrix}\ddot{\boldsymbol{\theta}}_r\\ \ddot{\boldsymbol{\theta}}_\delta\end{bmatrix}+\begin{bmatrix}\boldsymbol{H}_{11}&\boldsymbol{H}_{12}\\ \boldsymbol{H}_{21}&\boldsymbol{H}_{22}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}_r\\ \dot{\boldsymbol{\theta}}_\delta\end{bmatrix}+\begin{bmatrix}\boldsymbol{0}_{3\times1}\\ \boldsymbol{K}_\delta\boldsymbol{\theta}_\delta\end{bmatrix}=\begin{bmatrix}\boldsymbol{\tau}\\ \boldsymbol{0}_{4\times1}\end{bmatrix} \tag{7}$$

Where, $\boldsymbol{M}_{11}\in R^{3\times3}$, $\boldsymbol{M}_{12}\in R^{3\times4}$, $\boldsymbol{M}_{21}\in R^{4\times3}$ and $\boldsymbol{M}_{22}\in R^{4\times4}$ are the sub matrix of $\boldsymbol{M}(\boldsymbol{\theta}_{r\delta})$; $\boldsymbol{H}_{11}\in R^{3\times3}$, $\boldsymbol{H}_{12}\in R^{3\times4}$, $\boldsymbol{H}_{21}\in R^{4\times3}$ and $\boldsymbol{H}_{22}\in R^{4\times4}$ are the sub matrix of $\boldsymbol{H}(\boldsymbol{\theta}_{r\delta},\dot{\boldsymbol{\theta}}_{r\delta})$.

Further system dynamics Eqs. (7) can be written

$$\boldsymbol{M}_{11}\ddot{\boldsymbol{\theta}}_r+\boldsymbol{M}_{12}\ddot{\boldsymbol{\theta}}_\delta+\boldsymbol{H}_{11}\dot{\boldsymbol{\theta}}_r+\boldsymbol{H}_{12}\dot{\boldsymbol{\theta}}_\delta=\boldsymbol{\tau} \tag{8a}$$

$$\boldsymbol{M}_{21}\ddot{\boldsymbol{\theta}}_{\mathrm{r}} + \boldsymbol{M}_{22}\ddot{\boldsymbol{\theta}}_{\delta} + \boldsymbol{H}_{21}\dot{\boldsymbol{\theta}}_{\mathrm{r}} + \boldsymbol{H}_{22}\dot{\boldsymbol{\theta}}_{\delta} + \boldsymbol{K}_{\delta}\boldsymbol{\theta}_{\delta} = 0 \quad (8b)$$

$\boldsymbol{M}_{22}$ is the symmetric, positive definite and reversible matrix, the Eqs. (8) is expressed as

$$\ddot{\boldsymbol{\theta}}_{\delta} = -\boldsymbol{M}_{22}^{-1}(\boldsymbol{M}_{21}\ddot{\boldsymbol{\theta}}_{\mathrm{r}} + \boldsymbol{H}_{21}\dot{\boldsymbol{\theta}}_{\mathrm{r}} + \boldsymbol{H}_{22}\dot{\boldsymbol{\theta}}_{\delta} + \boldsymbol{K}_{\delta}\boldsymbol{\theta}_{\delta}) \quad (9)$$

Combine Eqs.(8) and Eqs.(9), the fully controllable form dynamic equation is expressed as

$$\boldsymbol{D}_1\ddot{\boldsymbol{\theta}}_{\mathrm{r}} + \boldsymbol{H}_1 = \boldsymbol{\tau} \quad (10)$$

Where, $\boldsymbol{H}_1 = \boldsymbol{H}_{11}\dot{\boldsymbol{\theta}}_{\mathrm{r}} + \boldsymbol{H}_{12}\dot{\boldsymbol{\theta}}_{\delta} - \boldsymbol{M}_{12}\boldsymbol{M}_{22}^{-1}(\boldsymbol{H}_{21}\dot{\boldsymbol{\theta}}_{\mathrm{r}} + \boldsymbol{H}_{22}\dot{\boldsymbol{\theta}}_{\delta} + \boldsymbol{K}_{\delta}\boldsymbol{\theta}_{\delta})$, $\boldsymbol{D}_1 = \boldsymbol{M}_{11} - \boldsymbol{M}_{12}\boldsymbol{M}_{22}^{-1}\boldsymbol{M}_{21}$. $\boldsymbol{D}_1$ is the symmetric, positive definite and reversible matrix.

Introducing the state vector $\boldsymbol{x}_1 = \boldsymbol{\theta}_{\mathrm{r}}$, $\boldsymbol{x}_2 = \dot{\boldsymbol{\theta}}_{\mathrm{r}}$, $\boldsymbol{x}_{\mathrm{d}} = [q_{\mathrm{d}0} \quad q_{\mathrm{d}1} \quad q_{\mathrm{d}2}]^{\mathrm{T}}$ is the augmented desired trajectory. Considering to external disturbance $\boldsymbol{\tau}_{\mathrm{d}} \in R^3$. Then the dynamic equation of the slow system(10) is expressed as in the following state-space representation

$$\begin{cases} \dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2, \\ \dot{\boldsymbol{x}}_2 = \boldsymbol{F}(x) + \boldsymbol{G}(x)\boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{d}}, \end{cases} \quad (11)$$

Where, $\boldsymbol{G}(x) = \boldsymbol{D}_1^{-1}$, $\boldsymbol{F}(x) = -\boldsymbol{G}(x)\boldsymbol{H}_1$.

According to the adaptive Backstepping method [15], this design method is a structural nonlinear controller, take the following variables

$$z_1 = \boldsymbol{x}_1 - \boldsymbol{x}_{\mathrm{d}} \quad (12)$$

$$z_2 = \boldsymbol{x}_2 - \dot{\boldsymbol{x}}_{\mathrm{d}} - \beta \quad (13)$$

Eqs.(12) of the time t derivative, combination Eqs.(13)

$$\dot{z}_1 = \dot{\boldsymbol{x}}_1 - \dot{\boldsymbol{x}}_{\mathrm{d}} = z_2 + \beta \quad (14)$$

And, $$\beta = -(k + \frac{1}{2})z_1 \quad (15)$$

β is the virtual control input. In addition, $\boldsymbol{z} = [z_1 \quad z_2]^{\mathrm{T}}$ is the definition of system performance output, meanwhile, it is also the evaluation index of the system, which is used to evaluate the performance of the nonlinear system.

The following algorithm for the space robot is proposed

$$\boldsymbol{\tau} = \frac{1}{\boldsymbol{G}(x)}(\boldsymbol{\tau}_{\mathrm{a}} + \boldsymbol{\tau}_{\mathrm{r}}) \quad (16)$$

$$\boldsymbol{\tau}_{\mathrm{a}} = -\boldsymbol{F}(x) + \ddot{\boldsymbol{x}}_{\mathrm{d}} + \dot{\beta} - z_1 - \boldsymbol{P}z_2 \quad (17)$$

$$\boldsymbol{\tau}_{\mathrm{r}} = -\frac{\gamma^2 + 1}{2\gamma^2}z_2 \quad (18)$$

Where, $\boldsymbol{\tau}_{\mathrm{a}}$ and $\boldsymbol{\tau}_{\mathrm{r}}$ is adaptive controllers and robust controllers, respectively. $\boldsymbol{P} = \mathrm{diag}[p_1 \quad p_2 \quad p_3]$ is the given evaluation coefficient, and the closed-loop system is uniformly ultimately bounded stable.

Combine Eqs. (11), Eqs. (13) and Eqs. (16)

$$\dot{z}_2 = \dot{\boldsymbol{x}}_2 - \ddot{\boldsymbol{x}}_{\mathrm{d}} - \dot{\beta} = -z_1 - pz_2 + \boldsymbol{\tau}_{\mathrm{r}} + \boldsymbol{\tau}_{\mathrm{d}} \quad (19)$$

Define, the L2 norm of interference signal $\boldsymbol{\tau}_{\mathrm{d}}(t)$ is expressed as

$$\|\boldsymbol{\tau}_{\mathrm{d}}(t)\|_2 = \left\{\int_0^{\infty} \boldsymbol{\tau}_{\mathrm{d}}^{\mathrm{T}}(t)\boldsymbol{\tau}_{\mathrm{d}}(t)\mathrm{d}t\right\}^{\frac{1}{2}} \quad (20)$$

Where, the L2 norm can measure the energy of $\boldsymbol{\tau}_{\mathrm{d}}(t)$.

In order to judge the interference suppression ability of the system, the performance index of the system is

$$J = \sup_{\|\boldsymbol{\tau}_{\mathrm{d}}\| \neq 0} \frac{\|z\|_2}{\|\boldsymbol{\tau}_{\mathrm{d}}\|_2} \quad (21)$$

Where, J is the L2 gain of the system, representative system robustness. The smaller the J indicates the better robust performance of the system.

L2 disturbance suppression problem can be equivalent to solve a dissipative inequality problem based on Lyapunov stability theory, namely to a positive number γ , γ is called interference suppression factor level, if there is a positive definite and differentiable function $V(x) \geq 0$ and

$$\dot{V} \leq 0.5(\gamma^2\boldsymbol{\tau}_{\mathrm{d}}^2 - \|z\|^2) \quad (\forall \boldsymbol{\tau}_{\mathrm{d}}) \quad (22)$$

Then $J \leq \gamma$. In order to obtain positive definite storage function $V(x)$ in Eqs.(22), it is needed to solve HJI(Hamilton-Jacobi Inequality). However, it is very difficult to solve the HJI equation of general nonlinear system. In order to make the system stability, usually used recursive Lyaponov function method to construct the storage function $\dot{V}$ of the system, that is negative.

Define Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \quad (23)$$

Combined Eqs.(14) and Eqs.(19), the derivative of the Lyapunov function is

$$
\begin{aligned}
\dot{V}_1 &= z_1(z_2+\beta)+z_2(\boldsymbol{\tau}_{\rm d}-z_1-pz_2-\boldsymbol{\tau}_{\rm r}z_2)\\
&=-(k+\frac{1}{2})z_1^2-pz_2^2+z_2(\boldsymbol{\tau}_{\rm d}-\boldsymbol{\tau}_{\rm r}z_2)\\
&\le -\frac{1}{2}z_1^2-\frac{1}{2}z_2^2-\frac{1}{2}(\frac{z_2}{\gamma}-\boldsymbol{\tau}_{\rm d})^2+\frac{\gamma^2}{2}\boldsymbol{\tau}_{\rm d}^2\\
&\le -\frac{1}{2}\|z\|^2+\frac{\gamma^2}{2}\boldsymbol{\tau}_{\rm d}^2
\end{aligned} \tag{24}
$$

The time integration of Eqs.(24) from 0 to ∞,

$$
2V_1(\infty)-2V_1(0)\le -\int_0^{\infty}\|z\|^2\,{\rm d}t+\gamma^2\int_0^{\infty}\boldsymbol{\tau}_{\rm d}^2\,{\rm d}t \tag{25}
$$

The system satisfied the dissipation inequality, that is, from the disturbance $\boldsymbol{\tau}_{\rm d}$ to the L_2 gain of the system performance, $J \le \gamma$. And by the formula (25), when $\boldsymbol{\tau}_{\rm d}=0$, the closed-loop system is asymptotically stable; when $\boldsymbol{\tau}_{\rm d}\ne 0$, the closed-loop system is uniformly ultimately bounded with the external disturbances are bounded.

4 SIMULATION EXAMPLES

To show the performance of the proposed controller, a simulation is carried out on a planar space robot system shown in Fig.1.

The actual plant parameters of the system are as follows: $l_0=1.5\,\rm m$, $l_1=l_2=3\,\rm m$, $m_0=200\,\rm kg$, $j_0=70\,\rm kg\cdot m^2$, $\rho_1=1.1\rm kg/m$, $\rho_2=1.1\rm kg/m$, $EI_1=100\rm N\cdot m^2$, $EI_2=50\rm N\cdot m^2$.

The control system parameters are selected as follows: $\mu=0.01$, $k=1$, $p_1=p_2=p_3=40$, $\gamma=1$.

Identify external disturbances as follows:

$$\boldsymbol{\tau}_{\rm d}=0.1\times[\mathrm{sgn}(\sin t)\quad \mathrm{sgn}(\cos t)\quad \mathrm{sgn}(\sin t)]^{\rm T}.$$

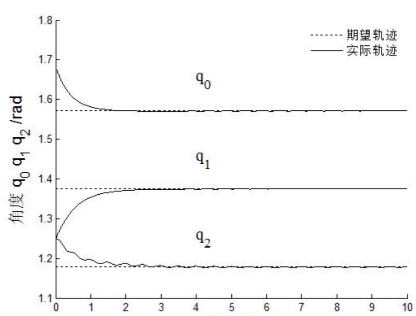

Fig.2 The comparison between the desired angular displacement and the actual one

The desired trajectories of links' joint angles are chosen as $q_{0\rm d}=\pi/2$, $q_{1\rm d}=7\pi/16$, $q_{2\rm d}=3\pi/8$, where the unit is in radians.

The initial states of the system are chosen as $q_0(0)=1.68$, $q_1(0)=1.25$, $q_2(0)=1.25$, where the unit is in radians.

The time taking in the simulation is $10\,\rm s$ .The controller performances are shown in Figs.2-4. Fig.2 is the comparison between the desired angular displacement and the actual ones of link joints. It can be seen from Fig.2 that good tracking performance is achieved through the application of the proposed controller. Fig.3 and Fig.4 are the vibration modes of flexible-link B_1 and flexible-link B_2.

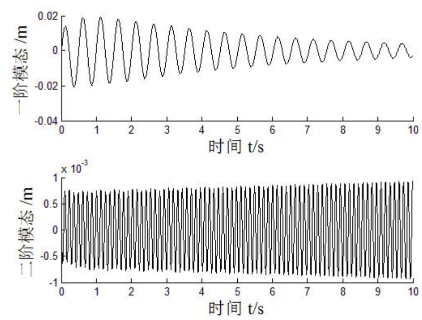

Fig.3 The vibration modes of flexible-link B_1

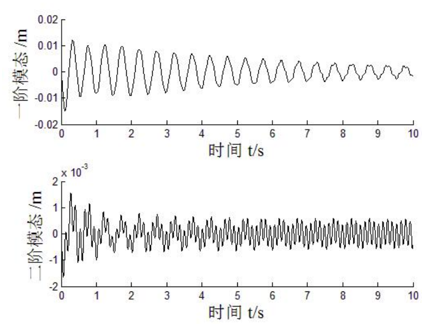

Fig.4 The vibration modes of flexible-link B_2

5 CONCLUSION

The dynamic model and trajectory tracking of two flexible links space robot with external bounded disturbance. Neglecting the microgravity, the dynamical model of the system is successively derived by combining with the system linear momentum, Lagrange equation and assumed mode method. A robust controller which makes the external bounded disturbances have the performance of the L-two-gain of the system was applied to control the system to track the desired trajectory; the closed loop system can restrain the external bounded disturbance signal. In the end, it can be extended to the multi flexible links space robot system. Numerical simulation results show that the proposed control algorithm can precision and stability control the joint tracking of the space robot with flexible links; the reliability and validity of the controller are proved.

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