FORM-FINDING OF CABLE NET STRUCTURE FOR LARGE MESH REFLECTOR

Xiaozhi Qi[1], Huaihu Zhang[2], Zhihuai Miao[3], *Bing Li[4]

[1]Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen, China, E-mail: ixiaozhiq@163.com

[2]Huawei Technologies Company Limited, Shenzhen, China, E-mail: zhanghuaihu_1990@163.com

[3]Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen, China, E-mail: miaozhihuai@hitsz.edu.cn

[4]Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen, China, E-mail: libing@hitsz.edu.cn

ABSTRACT

This paper presents a dual-objective optimization form-finding model of a mesh cable net that ensures both appropriate surface accuracy and uniform tension. The approach is based on the force density method with considering a sag-to-span ratio or not. A group of the optimal force density which meets the requirements of force uniformity and antenna accuracy can be found by using genetic algorithm to optimize. In order to improve the accuracy of form-finding, a modified form-finding method combining the FDM with considering boundary deformation and parabolic geometric constraint is presented in this paper. By the modified form-finding method, several specific examples are computed and the results show that the method can get better surface accuracy and force uniformity and be capable of high computational efficiency. Then the iterative form-finding method with boundary deformation and its influence on the computational accuracy are also discussed. Meanwhile, the effects of different conditions on the cable structural rigidity are presented in the final.

Keywords: form-finding, cable net, force density method, mesh reflector

1 INTRODUCTION

With the development of satellite-communication and earth-observation technologies, the demands for large and light space-deployable antennas have become more and more urgent [1]. Large mesh reflectors are widely used in large aperture space antenna systems because they are lightweight, and can be packaged compactly and easily [2]. This mesh is stretched over a cable net, usually made of stiff unidirectional composite filaments and attached to a deployable truss. Therefore, the accuracy of mesh reflector depends on the shape of the cable net. It is extremely important to study the form-finding of the cable net structure [3]. The force density method (FDM) was first proposed by Schek to find forms of architectural structure without solving any nonlinear equations [4]. In the last twenty years, FDM and its improvements have been used widely in form-finding of tensegrity structures due to its validity [5]. The authors' research group has developed a method to calculate the cable net, which combines the original force density method and the parabolic surface constraint, and the genetic algorithm was applied to ensure that each cable reaches a desired tension effectively [6]. However, traditionally, the form-finding methods are used in the hypothetical condition that the support truss is fully constrained and has no boundary deformation. The actual boundary of cable net in contact with the truss will be deformed when preload, so that the computational results are not actually accurate. Even a small amount of deformation will make the force performance and the surface accuracy of the cable net produces a large of influence. In severe cases, there exists some slack of cable segment, which leads to failure of the antenna [7]. As is shown in Figure 1, we display the study objective.

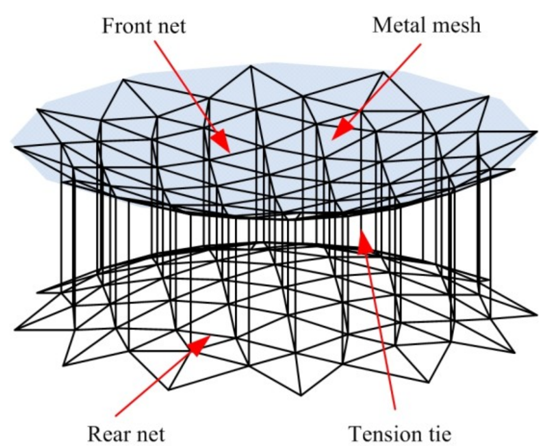


Figure 1 Composition of the cable net reflector

This paper firstly gives a brief analysis of the meshing for cable net structure antenna, then dual-objective optimization form-finding based on the force density method is presented. At the same time, boundary deformation of cable net is also taken in to consideration. We use three-dimensional grid as form classification. Root mean square error based on net surface accuracy and force uniformity are taken as the double objective optimization method, then research the form-founding problem by considering a sag-to-span ratio or not in contrast. Through the establishment of cable net structure finite element analysis model, we explore an effective method to adjust the accuracy of cable net structure, next, carry out the analysis about the accuracy influence of different vertical line adjustment, then optimization mathematical model of cable net adjustment is established.

The rest part of this paper is organized as follows: firstly, the basic principle of force density method is discussed in Section 2. Secondly, we build the form-finding model of dual-objective optimization for space cable net structure in Section 3. In Section 4, one case of symmetric parabolic cable net is simulated and analyzed. In Section 5, the iterative form-finding method with boundary deformation is also discussed. Conclusions are drawn in the last section in this paper.

2 BASIC PRINCIPLE OF FORCE DENSITY METHOD

The design of the cable net systems is very different from other structures. For cable net, the shape depends on the boundary conditions and suffered tension. Without tension, the cable net does not have stiffness, also can't bear the load. Force density method is now widely used in form-finding for cable net structure. This method introduces the force density into the cable net equilibrium equations, which turns the nonlinear equations into linear equilibrium equations. It's a very efficient method for form-finding of space cable net.

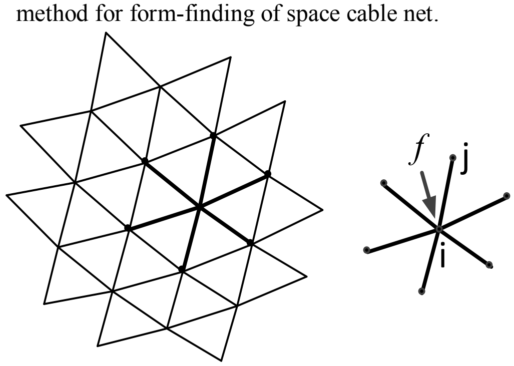


Figure 2 Part of a cable net

As is shown in Figure 2, the force equilibrium equations for a node in the tension cable net structure can be written as follows:

$$\begin{cases} \sum \frac{T_{ij}}{L_{ij}}\left(x_i - x_j\right) = f_x \\ \sum \frac{T_{ij}}{L_{ij}}\left(y_i - y_j\right) = f_y \\ \sum \frac{T_{ij}}{L_{ij}}\left(z_i - z_j\right) = f_z \end{cases} \quad (1)$$

T_{ij} : Force of the *ij* cable segment

L_{ij} : Length of the cable segment

x, y, z : Spatial coordinates of the cable segment

f : External force.

By introducing the force density, $q_{ij} = \left(T_{ij}/L_{ij}\right)$, equilibrium equations are transformed into the linear equations:

$$\begin{cases} \sum q_{ij}\left(x_i - x_j\right) = f_x \\ \sum q_{ij}\left(y_i - y_j\right) = f_y \\ \sum q_{ij}\left(z_i - z_j\right) = f_z \end{cases} \quad (2)$$

For the form-finding of cable net by the force density method, with the given force density of each cable segment, boundary conditions and external stress situation, the nodal coordinates of cable net system can be easily solved by linear equations (2). Thereby, a cable net shape can be determined. Meanwhile, in order to facilitate the calculation program, we introduce the structural topology matrix C into the equilibrium equations. All nodes' force balance equations in the form of matrix are established:

$$\begin{cases} C^T QCX = F_x \\ C^T QCY = F_y \\ C^T QCZ = F_z \end{cases} \quad (3)$$

Where, **C** is the geometric topology matrix of cable net structure; **Q** is the diagonal matrix constituted by force density value of each cable segment for the cable net structure; F_x , F_y , F_z are the column vectors of external force along the corresponding coordinate direction for each node. The equation (3) is solved to obtain spatial coordinate of each node and the length L_{ij} of each cable can be calculated. The tension of cable segment can be determined by the formula $T_{ij} = q_{ij} \times L_{ij}$ .With the given cross-sectional area A and the elastic modulus E , the original length of the cable segment can be obtained by the formula $L_{ij}^0 = L_{ij} \Big/ \left[1 + T_{ij} / \left(EA\right)\right]$.

3 FORM-FINDING MODEL OF DUAL-OBJECTIVE OPTIMIZATION

Form-finding of cable net antenna, which is required to find a group of optimal pre-tensions, makes the entire cable net in equilibrium state, meanwhile, ensures both the surface accuracy requirements and uniform tension of the cable segment. But the force density method can't guarantee uniform force of all cable net segments.

Considering the above factors, the form-finding model takes the force density in each cable segment as design variable, uses the ratio of force uniformity and surface accuracy as dual-objective to establish the optimization model. Then find a group of optimal force densities that meets two requirements of force uniformity and surface accuracy. Optimization model is set up as follows:

$$
\begin{aligned}
&\text{Find} \quad q_1, q_2, \cdots q_m \\
&Min \quad f = \omega_1 \bar{f}_1 + \omega_2 \bar{f}_2 \\
&f_1 = \sqrt{\left(\sum_{i=1}^{n} \delta_i^2\right) / n}; \; f_2 = \frac{T_{\max}}{T_{\min}} \\
&\bar{f}_i = \frac{f_i - f_i^{\min}}{f_i^{\max} - f_i^{\min}} \quad i = 1 \sim 2 \\
&S.t. \quad \mathbf{C}^{\mathrm{T}}\mathbf{QC}X = F_{\mathbf{x}}; \\
&\qquad \mathbf{C}^{\mathrm{T}}\mathbf{QC}Y = F_{\mathbf{y}}; \\
&\qquad \mathbf{C}^{\mathrm{T}}\mathbf{QC}Z = F_{\mathbf{z}} \\
&\qquad 0 < q_i \le [q]
\end{aligned}
\tag{4}
$$

In the formula, the designed variable q_i is the force density value of each cable segment, m is the number of cable segments; f is the objective function, ω_1 and ω_2 are the target weights, here $\omega_1 = \omega_2 = 0.5$, f_1 is taken as a cable net surface accuracy, δ_i is the coordinate error between node i on cable net and the ideal node, n is the free node number of the cable net, f_2 is the ratio of largest cable tension of the segment with minimum cable tension of segment, $T_{\max}$ is the largest cable tension of segment, $T_{\min}$ is the minimum tension, $\bar{f}_i$ is the dimensionless normalization function; constraint conditions are the force equilibrium equations, $\mathbf{C}$ is the geometric topology matrix of cable net system, $\mathbf{Q}$ is the diagonal matrix constituted by force density value of each cable segment for the cable net structure; X, Y, Z are coordinate column vectors of the nodes; F_x, F_y, F_z are the column vectors of external force along the corresponding coordinate direction for each node.

The form-finding of cable net is based on multi-pivot six ring cable net antenna in this paper, whose front and rear net are symmetrical parabolic, as is shown in Figure 3. Cable net system contains 403 cable segments, 134 nodes, where, nodes on the boundary are fixed on peripheral truss, which is assumed to be fully constrained.

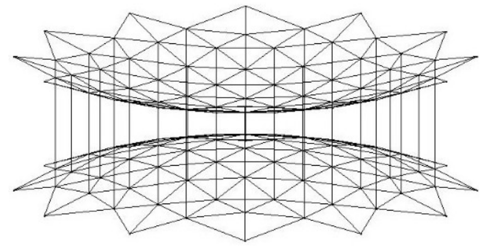
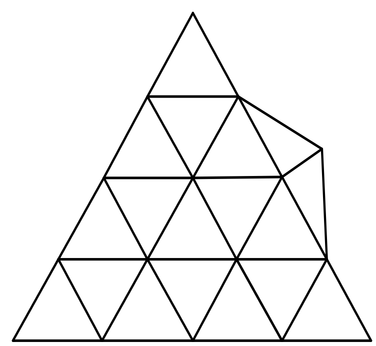
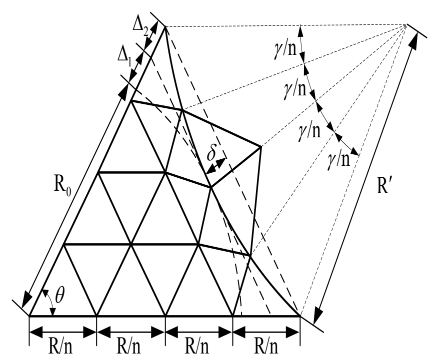


Figure 3 Multi-pivot six ring cable net

Since the cable net is centrosymmetric structure, we take 1/6 unit of cable net as the research objective to perform form-finding analysis, and consider form-finding in both cases with sag-to-span or not. As is shown in Figure 4(a): the cable mesh generation without sag-to-span cable net and in Figure 4(b): taking into account the effect of the boundary with the introduction of sag-to-span. R is the radial radius of the cable net, R_0 is the valid reflection radius of cable net, δ is the ratio of sag-to-span, outer nodes are distributed on the aliquots arc with the radius R' and the opening angle γ .



(a) Mesh form of Irrespective of span ratio



(b) Mesh form under consideration of span ratio

Figure 4 Cable net mesh schematic

Sag-to-span ratio is calculated as:

$$\rho = \frac{\delta}{2R_0 \tan(\theta / 2)} \tag{5}$$

The relationship between R and R_0 is

$$\frac{R}{R_0}=\frac{1+2\rho\tan(\theta/2)}{\cos(\theta/2)} \tag{6}$$

The radius of the arc R′ and the opening angle γ are calculated respectively:

$$R'=\frac{\delta^2+R^2\sin^2(\theta/2)}{2\delta} \tag{7}$$

$$\gamma=2\arccos\left(\frac{R'-\delta}{R'}\right) \tag{8}$$

We also choose 10% as sag-to-span ratio for cable net mesh generation, and obtain coordinates of outer space nodes by the above equations. Figure 5 shows the serial number of the 1/6 cable net node and cable segment, where, the boundary nodes 14,15,16 are fixed, and the others are free nodes. The structural and material parameters of cable net are as follows: cable net diameter D=3882mm, border node distance between front net with rear net H=1190mm, focal length F=2500mm, radius of the net cable segment R=0.5mm, the elastic modulus of the material $E=1.24\times10^{11}N/m$.

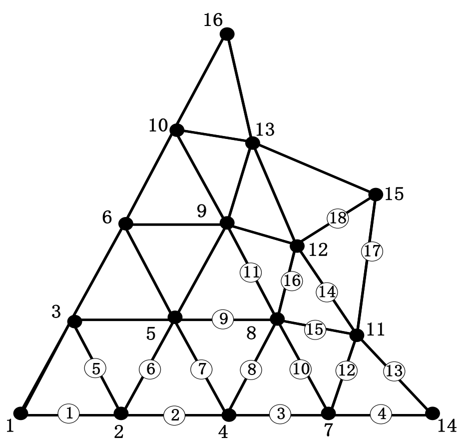


Figure 5 Cable node and cable segment Number

4 SIMULATION CASE

By the aforementioned form-finding optimization model, we can use MATLAB programming solver to find a set of optimal force density values to the cable net with sag-to-span or not. Then obtain spatial coordinates for each node, cable segment tension and surface accuracy under this group of tension.

4.1 Form-finding without sag-to-span

After form-finding without considering sag-to-span, cable net node Z coordinate and corresponding longitudinal pretension T are recorded in Table 1. Due to the symmetry of the cable net, the values listed in the table only include eight free nodes, and $Z_i^{'}$ is the ideal coordinates, δ_{rms} is the surface accuracy. Analysis of data in this table shows that the overall tension of longitudinal cable is evenly distributed in internal cable net, the value 3.01N of the outer node 11 is the maximum tension, minimum tension appeared on the 12th node is 1.30N. Table 2 shows the tension of the front net cable segment. Due to the symmetry, we only record 18 cable segments shown in Figure 5. The data shows that the tension of internal cable segment maintains a high uniformity, but the force distribution of edge cable is very uneven and quite different. Tensions of edge segment 4,13,17 are larger, where, the maximum tension appears in cable segment 13 with 24.55N, and tension in cable segments 12, 14 are small, where, the minimum tension appears in cable segment 14 with 2.43N. So, this kind of force distribution is prone to produce local stress concentration, and affect the force performance of the whole cable net structure. And from Table 1, the overall net surface accuracy with just 0.45mm is not high.

Table 1 Z coordinates and tension of vertical cable after form-finding without sag-to-span

Node No.	$Z_i^{'}$ (mm)	Z_i (mm)	T_k (N)	δ_{rms} (mm)
1	218.33	218.33	2.94	
2	241.85	241.85	2.63	
4	312.41	312.40	2.62	
5	288.92	288.91	2.43	
7	430.23	430.40	2.57	0.451
8	383.09	383.07	2.61	
11	524.37	524.13	3.01	
12	500.83	500.70	1.30	

Table 2 Tension of each cable segment after form-finding without sag-to-span

Segment	Force (N)	Segment	Force (N)	$\frac{T_{max}}{T_{min}}$
1	10.16	10	9.95	
2	10.42	11	10.67	
3	10.68	12	2.47	
4	18.92	13	**24.55**	
5	8.54	14	**2.43**	10.10
6	8.40	15	9.50	
7	8.40	16	7.74	
8	8.39	17	21.20	
9	8.52	18	13.87	

4.2 Form-finding with sag-to-span

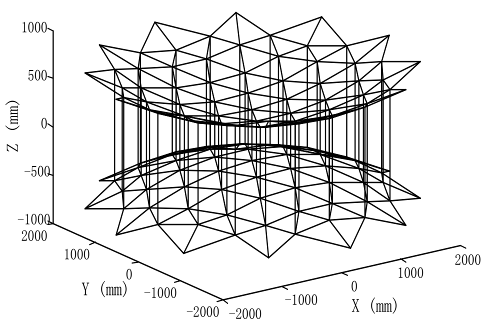
With considering sag-to-span, the Z coordinates and tension of vertical cable can be computed and the results are in Tables 3 and 4. From the data in Table 3, it can be seen that the internal tension distribution of longitudinal cable is more uniform than the case without considering sag-to-span after optimized form-finding. Due to analyses of the cable tension data in Table4, it can be shown that although the forces on sub marginal cable segment 4,13,17,18 are still very large, but the ratio of the maximum and minimum tension on the overall net is 2.38, in Table 2, the ratio of maximum and minimum tension is 10.10. The cable net tension maintains high uniformity, and force performance of cable net surface is better. From Table 3, the net surface accuracy with 0.216mm is also improved. A three dimensional cable net is plotted according to the cable net structure with node coordinates after form-finding, as is shown in Figure 6.

Table 3 Z coordinates and tension of vertical cable with sag-to-span in form-finding

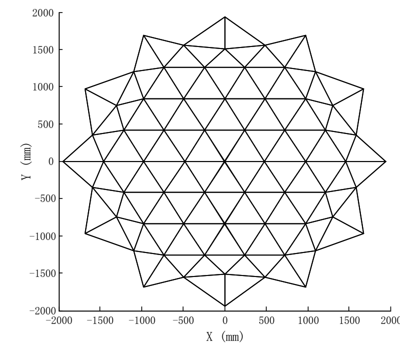
Node No.	Z_i' (mm)	Z_i (mm)	T_k (N)	δ_{rms} (mm)
1	218.33	218.33	3.95	
2	241.85	241.87	3.48	
4	312.41	312.49	3.64	
5	288.92	288.95	3.33	
7	430.23	430.21	4.12	0.216
8	383.09	383.12	3.34	
11	483.00	483.07	3.60	
12	445.36	445.36	3.03	

Table 4 Tension of each cable segment after form-finding with sag-to-span

Segment	Force (N)	Segment	Force (N)	$\frac{T_{\max}}{T_{\min}}$
1	13.60	10	15.99	
2	13.87	11	11.55	
3	15.44	12	10.67	
4	24.70	13	**25.39**	
5	11.21	14	**10.66**	2.38
6	11.11	15	10.69	
7	12.24	16	15.98	
8	11.04	17	17.11	
9	11.27	18	21.72	



(a) Three angles figure



(b) A plan view

Figure 6 Three dimensional cable net after form-finding

After form-finding through MATLAB, we import the pre-tension as the initial strain of cable net, then establish an ANSYS finite element model. the analysis shows that the order of the maximum displacement deformation is 10-15, deformation nephogram shown in Figure 7 also indicates that the group of cable net is in a state of equilibrium.

Pre-stressed configuration obtained by this optimization method is correct. In summary, considering sag-to-span of cable net form-finding, it is possible to obtain a more uniform tension as well as better accuracy by the form-finding method. However, these results are based on the boundary nodes which are fully constrained, form-finding of the cable net with boundary deformation will be discussed in the following section.

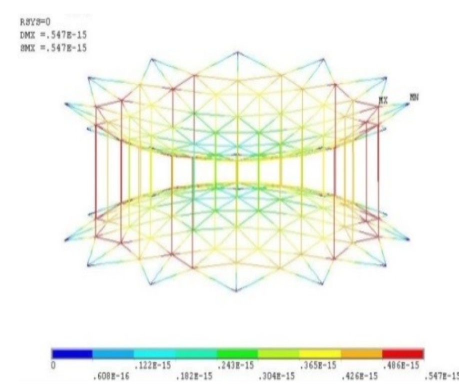


Figure 7 Deformed cloud of cable net system under pre-stressed

5 FORM-FINDING ANALYSIS OF CABLE NET WITH BOUNDARY DEFORMATION

As is mentioned before, the calculations to form-finding for the whole cable net is under the assumption that its boundary is fully constrained. The actual cable net boundary contacted with the truss will be deformed when preload, so that the results are actually not accurate. The truss is pulled by the cable net, then boundary deformation will occur, even a small amount of deformation will make a large effect on the force performance and the surface accuracy of the cable ne. In severe case, there may exist some slack cable segment, which leads to a failure of the cable net antenna and quit of work.

Taking into account the size and weight limited by space launch vehicles, we can't improve the stiffness by increasing the size of the truss element structure, which means that the effect of boundary deformation is ignored. As long as the deformation of the truss is in a permissive scope, it is feasible that the deformed shape cable net can meet the requirements of accuracy and stress.

First, take the obtained cable net shape after form-finding with fully constrained as the initial state for analysis. When the boundary forces of border boundary cable net are applied on the deployable truss, the boundary nodes are deformed and stretched under the cable net tension, border coordinates will change, which means boundary conditions of the cable net form-finding also change. Therefore, in the process of form-finding, internal cable net is to be updated and boundary results of the force are to be constantly updated as well. Iterative process will not stop until the changes of border force or boundary coordinate are smaller than a convergence value. The block diagram of iterative process after form-finding is considered in the case of boundary deformation, as is shown in Figure 8.

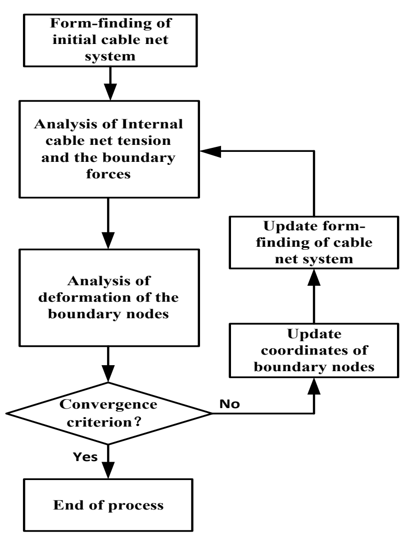


Figure 8 Iterative process of form-finding with boundary deformation

Considering that the initial boundary deformed is 0.2mm, the boundary reaction force meets the convergence conditions ($\Delta F \leq 0.01$) after eight iterations, then stop the iteration, the surface accuracy $\delta_{rms} = 0.198mm$, the corresponding boundary node circumferential deformation of border truss is 0.212mm. Extract the values of Z coordinates and forces of longitudinal tension ties after form-finding, which are recorded in Table 5, the force of each cable segment on the front net is recorded in Table 6 below. Compared to the results with considering sag-to-span, the overall force and tension uniformity are basically the same. The cable net surface accuracy slightly increases. The results are closer to the case of pre-stressed cable net in actual circumstance and the deformation of the cable net boundary node is also considered. So the accuracy effects due to truss deformation can be ignored for this group of tension configuration in the cable net system. In this case, form-finding is directly based on tension configuration and required mesh accuracy under the presence of deformation.

Table 5 Z coordinates and tension of vertical cable after form-finding with boundary deformation

Node No.	Z_i' (mm)	Z_i (mm)	T_k (N)	δ_{rms} (mm)
1	218.33	218.33	3.86	
2	241.88	241.87	3.70	
4	312.54	312.49	3.43	
5	288.98	288.95	3.41	0.198
7	430.29	430.21	4.31	
8	383.19	383.12	3.40	
11	483.04	483.07	3.73	
12	445.36	445.36	3.12	

Table 6 Tension of each cable segment after form-finding with boundary deformation

Segment	Force (N)	Segment	Force (N)	$\frac{T_{max}}{T_{min}}$
1	13.26	10	16.86	
2	13.88	11	10.93	
3	14.68	12	12.13	
4	23.72	13	**25.37**	
5	12.57	14	**10.48**	2.42
6	12.14	15	10.48	
7	10.93	16	16.82	
8	10.49	17	18.66	
9	12.31	18	22.97	

6 CONCLUSION

This article gives an effective implementation of form-finding of cable net system, comparatively studies the effects of surface accuracy and tension uniformity of the structure in form-finding with sag-to-span or not. ANSYS model verifies the effectiveness of this method. Considering the truss structure and deformation of the contact boundary node, we propose an efficient iterative process, which can quickly generate the boundary deformation and form-finding results. The results are meaningful for the actual cable design and engineering application.

Acknowledgement

This work was financially supported by the State Key Laboratory of Robotics and System (HIT) (SKLRS201508B), in part by Shenzhen Research Funds (JCYJ20150529141408781 and JCYJ20140417172417129).

References

[1] Puig L, Barton A and Rando N (2010) A review on large deployable structures for astrophysics missions. Acta Astronautica 67:12-26.
[2] Santiago-Prowald J and Baier H (2013) Advances in deployable structures and surfaces for large apertures in space. CEAS Space Journal 5:89-115.
[3] Morterolle S, Maurin B, Quirant J and Dupuy C (2012) Numerical form-finding of geotensoid tension truss for mesh reflector. Acta Astronautica 76:154-163.
[4] Schek H (1974) The force density method for form finding and computation of general networks. Computer Methods in Applied Mechanics and Engineering 3:115-134.
[5] Tibert A and Pellegrino S (2011) Review of form-finding methods for tensegrity structures. International Journal of Space Structures 26(3): 241-256.
[6] Chu Z, Deng Z, Qi X and Li B (2014) Modeling and analysis of a large deployable antenna structure. Acta Astronautica 95:51-60.
[7] Ando K, Mitsugi J and Senbokuya Y (2000) Analyses of cable-membrane structure combined with deployable truss. Computer and Structures 74:21-39.