

BASED ON ADAPTIVE OBSERVER SLIDING MODE CONTROL OF FREE-FLOATING FLEXIBLE-JOINTS SPACE ROBOT

*Lijiao Zhang, Li Chen

School of Mechanical Engineering and Automation, Fuzhou University, Fujian, China, E-mail: lijiao@126.com

ABSTRACT

The problem of trajectory tracking and vibration suppression control for free-floating flexible-joints space robot system with external disturbance were discussed. The dynamical equation of the free-floating space robot with flexible joints is successively derived by combining with the system linear momentum and angle momentum conservation, Lagrange equation, the assume mode method. Introduced a joint flexibility compensation controller to improve the equivalent stiffness of the joint, then used the singular perturbation method, and the system was decomposed into a slow subsystem which was described joint rigid trajectory tracking and a fast subsystem which was described the vibration of flexible joints. For the slow subsystem, proposed the based on adaptive observer sliding mode control method for flexible-joints space robot, designed an observer to monitor the state variables of flexible-joints space robot system, and applies sliding mode variable structure to design the controller. For the fast subsystem, used the speed difference between the feedback control law to suppress the elastic vibration of flexible joints which was caused by system, guarantee the stability of the system. Numerical simulation results show that the proposed integrated control algorithm efficiency.

1 INTRODUCTION

With the development of space technology, the requirements of the space robot are getting higher and higher, the application of flexible space robot is becoming more and more extensive [1]. The traditional robot dynamics system has been unable to meet the requirements of dynamic analysis and control of flexible space robot. Space robot with planetary gear and application of the lightweight space manipulators has a large joint flexibility, the joint flexibility can cause the rotation error between the rotating angle of the mechanical arm and the rotation angle of the drive motor, and the accuracy and the stability of the control system are affected. At the same time, the joint flexibility will lead to the vibration of the manipulator in the process of movement [2]. The accuracy and stability of the motion control of space robot will be greatly affected if the joint flexibility is neglected in the tracking control of the robot system [3]. Therefore, how to establish a dynamic model and design a high

performance controller to control the flexible joint is a problem that must be faced and solved in the research and application of space robot [4-8].

The control design is heavily connected to the Prescribed Performance Control approach introduced in [9] that was proposed to design robust controllers capable of guaranteeing output tracking with pre-scribed performance. Kofigar proposed an adaptive algorithm with simplicity and universality properties to ensure robust tracking, and the parameters of the robot system and the external disturbance are considered, but the effect of joint flexibility on the robot control is not considered [10]. Chen et al. studied the dynamic modeling and singular perturbation control of free-floating space robot with normal flexible joints and introduced a joint compensator to accommodate the equivalent joint flexibility of the joints, but the unknown parameters aren't considered [11].

In order to achieve the asymptotic tracking of the free-floating flexible-joint space robot and suppression the flexible vibration caused by the joint flexibility, Used the singular perturbation method, and the system was decomposed into a slow subsystem which was described joint rigid trajectory tracking and a fast subsystem which was described the vibration of flexible joints. First, to eliminate the limitation of joint flexibility for applications of the traditional singular perturbation control technique in space robot with normal flexible joints, a joint flexibility compensator was introduced, which can level down the equivalent joint flexibility of the system. For the slow subsystem, proposed the based on adaptive observer sliding mode control method for flexible-joints space robot, designed an observer to monitor the state variables of flexible-joints space robot system, and applies sliding mode variable structure to design the controller. For the fast subsystem, used the speed difference between the feedback control law to suppress the elastic vibration of flexible joints which was caused by system, guarantee the stability of the system.

2 DYNAMIC MODEL OF FREE-FLOATING FLEXIBLE-JOINT SPACE ROBOT

The structure of free-floating flexible-joint space robot of a rigid body satellite, two rigid links and two flexible-joints is shown in Fig.1. The links were

connected in serial and were actuated by individual actuators. O_1 is the rotational center of the revolute joint between B_1 and B_0 . O_2 is the symmetrical axis of each link.

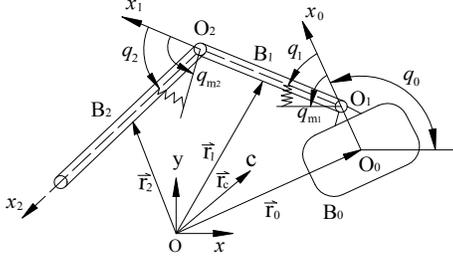


Fig.1 A planar free-floating flexible-joint space manipulator

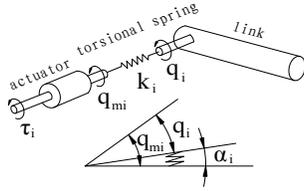


Fig.2 Simple model of a flexible joint

Establishment the inertial coordinate system (OXY) and the link coordinate system (O_i, x_i, y_i) , The i th flexible joint, in which the actuator rotor is directly coupled to its link, is shown schematically in Fig.2. Each joint was dynamically simplified as a linear tensional spring that works as a connector between the actuator rotor and the link. Referring to Fig.2, let $\mathbf{q}_m = [q_{m1}, q_{m2}]^T$ be the vector denoting the angular displacements of the actuator shaft angles, where the vector denoting the link angular displacements is $\mathbf{q}_0 = [q_1, q_2]^T$, q_0 is attitude angle of the base.

Therefore, in the design of controller, the effect of joint flexibility must be considered, so as to ensure the stability of the space robot. Then by the linear momentum conservation, the Euler-Lagrange dynamic equations of the flexible-joint space manipulator are

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_0 + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}_0, \dot{\mathbf{q}}_m) = \mathbf{u}_0 \quad (1)$$

$$\mathbf{J}\ddot{\mathbf{q}}_m + \mathbf{u}_0 = \mathbf{u} \quad (2)$$

$$\mathbf{u}_0 = \mathbf{K}(\mathbf{q}_m - \mathbf{q}_0) \quad (3)$$

Where, $\mathbf{q} = [q_0, \mathbf{q}_0^T]^T$, $\mathbf{M}(\mathbf{q}) \in R^{2 \times 2}$ and $\mathbf{J} \in R^{2 \times 2}$ are the links and actuators inertia positive-definite matrices, respectively; $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}_0, \dot{\mathbf{q}}_m) \in R^{2 \times 1}$ is the Coriolis or centrifugal matrix; $\mathbf{u}_0 \in R^{2 \times 1}$ is the joint torque/force delivered by actuators; $\mathbf{u} \in R^{2 \times 1}$ is the

vector of elastic torques at joints; $\mathbf{K} = \text{diag}[k_1 \ k_2]$ is the joint stiffness constant matrix.

3 DESIGN THE CONTROLLER

3.1 Control System Model Based on Joint Flexible Compensation

Examples are provided for different kinds of reference. The space robot system with free-floating flexible-joint, which is established by Eqs.(1), (2) and (3). The traditional singular perturbation approach is thus applicable to robots whose joint stiffness is large enough [12]. But the joint stiffness of the actual space robot is always not large enough, a flexibility compensator proposed by Liu et al. [13] is introduced.

Define \mathbf{u} as

$$\mathbf{u} = \mathbf{u}_m + \mathbf{K}_n \mathbf{u}_n \quad (4)$$

Where, $\mathbf{u}_m = -\mathbf{K}_c \mathbf{u}_0$ is a flexibility compensator, $\mathbf{K}_c \in R^{2 \times 2}$ is a positive-definite flexibility compensation diagonal matrix, define $\mathbf{K}_n = \mathbf{K}_c + \mathbf{I}$, \mathbf{I} is a 2-order identity matrix, $\mathbf{u}_n \in R^{2 \times 1}$ is a new controller.

Then combination Eqs.(2), (3) and (4), Eqs.(5) is derived

$$\mathbf{J}\mathbf{K}_c^{-1}\ddot{\mathbf{u}}_0 + \mathbf{u}_0 = \mathbf{u}_n - \mathbf{K}_n^{-1}\mathbf{J}\ddot{\mathbf{q}}_0 \quad (5)$$

Where, $\mathbf{K}_c = \mathbf{K}\mathbf{K}_n$ is the equivalent joint stiffness. Obviously, \mathbf{K}_c can be adjusted to any desired value as long as the reasonable \mathbf{K}_c is taken.

By the singular perturbation approach, the system is transformed into two subsystems, as low subsystem and a flexible-joint fast subsystem. The control law \mathbf{u}_n is decomposed into $\mathbf{u}_{ns} \in R^{2 \times 1}$ and $\mathbf{u}_{nf} \in R^{2 \times 1}$

$$\mathbf{u}_n = \mathbf{u}_{ns} + \mathbf{u}_{nf} \quad (6)$$

Define singular perturbation positive proportional factor ε , define $\mathbf{K}_e = \mathbf{K}_1/\varepsilon^2$, $\mathbf{K}_1 \in R^{2 \times 2}$ is a positive-definite diagonal matrix. Eq.(7) is the controller for the fast subsystem

$$\mathbf{u}_{nf} = -\varepsilon \mathbf{K}_2 \dot{\mathbf{u}}_0 \quad (7)$$

And Eq.(5) is transformed into

$$\varepsilon^2 \mathbf{J}\ddot{\mathbf{u}}_0 + \varepsilon \mathbf{K}_1 \mathbf{K}_2 \dot{\mathbf{u}}_0 + \mathbf{K}_1 \mathbf{u}_0 = \mathbf{K}_1 (\mathbf{u}_{ns} - \mathbf{K}_n^{-1} \mathbf{J}\ddot{\mathbf{q}}_0) \quad (8)$$

Where, $\mathbf{K}_2 \in R^{2 \times 2}$ is a positive-definite diagonal matrix.

ε is a very small positive proportion factor. And the stiffness coefficient of flexible joints $\mathbf{K} \rightarrow \infty$. Then

the flexible joint is equivalent to rigid. Then combination Eqs.(1) and(8), the dynamics of the slow subsystem model of flexible-joint space robot is derived

$$\mathbf{M}_s(\mathbf{q})\ddot{\mathbf{q}}_0 + \mathbf{H}_s(\mathbf{q}, \dot{\mathbf{q}}_0) = \mathbf{u}_{ns} \quad (9)$$

Where, $\mathbf{M}_s(\mathbf{q}) = \mathbf{M}(\mathbf{q}) + \mathbf{K}_n^{-1}\mathbf{J}$ is positive definite matrix, $\mathbf{H}_s(\mathbf{q}, \dot{\mathbf{q}}_0)$ is a simplified column vector of $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}_0, \dot{\mathbf{q}}_m)$.

The proposed integrated control algorithm \mathbf{u} can precision and stability control the joint tracking of the space robot with flexible joints, and active vibration suppression of flexible joints. The flexible-joint fast subsystem \mathbf{u}_{nf} represents the flexible part and the slow subsystem \mathbf{u}_{ns} represents the rigid part.

3.2 Design Controller for the Slow Subsystem

Introducing the state vector $\mathbf{x}_1 = \mathbf{q}_0$, $\mathbf{x}_2 = \dot{\mathbf{q}}_0$, $\mathbf{x} = [\mathbf{x}_1 \quad \mathbf{x}_2]^T$, the augmented desired trajectory is $\mathbf{x}_d = [q_{d1} \quad q_{d2}]^T$, Tracking error and its derivative are $\mathbf{e} = \hat{\mathbf{x}}_1 - \mathbf{x}_d$, $\dot{\mathbf{e}} = \hat{\mathbf{x}}_2 - \dot{\mathbf{x}}_d$. Considering to external disturbance $d(t) \in R^2$, the dynamic equation of the slow system(9) is expressed as in the following state-space representation

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + b[f(x) + g(x)\mathbf{u}_{ns} + d(t)] \\ \mathbf{y} = \mathbf{C}^T\mathbf{x} \end{cases} \quad (10)$$

Where, $g(x) = \mathbf{M}_s^{-1}(\mathbf{q})$, $f(x) = -\mathbf{M}_s^{-1}(\mathbf{q})\mathbf{H}_s(\mathbf{q}, \dot{\mathbf{q}}_0)$,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Design observer as follows

$$\begin{cases} \dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + b[\hat{f}(\hat{\mathbf{x}}) + \hat{g}(\hat{\mathbf{x}})\mathbf{u}_{ns} - \nu_1 - \nu_2] + K(\mathbf{y} - \mathbf{C}^T\hat{\mathbf{x}}) \\ \hat{\mathbf{y}} = \mathbf{C}^T\hat{\mathbf{x}} \end{cases} \quad (11)$$

Where, $\hat{\mathbf{x}}$ is the estimated value of \mathbf{x} , $K = [K_1 \quad K_2]^T$, K is the gain of observer, $\hat{f}(\hat{\mathbf{x}})$ and $\hat{g}(\hat{\mathbf{x}})$ are the estimated value of $f(x)$ and $g(x)$, $\nu(t)$ is the robust term.

According to the observer Eqs.(11), $f(x)$ and $g(x)$ use neural network to estimate, unknown continuous nonlinear function can combine ideal weight \mathbf{W}^* and basis functions $\mathbf{h}(x)$ to express

$$f(x) = \mathbf{W}_1^* \mathbf{h}_1(x) + \varepsilon_1(x), \varepsilon_1(x) \leq \varepsilon_{1,N} \quad (12)$$

$$g(x) = \mathbf{W}_2^* \mathbf{h}_2(x) + \varepsilon_2(x), \varepsilon_2(x) \leq \varepsilon_{2,N} \quad (13)$$

Where, $\varepsilon_1(x)$ and $\varepsilon_2(x)$ are the neural network approximation error.

Hypothetical ideal weight \mathbf{W}_1^* and \mathbf{W}_2^* are bounded as follows

$$\|\mathbf{W}_i^*\|_{i,F} \leq \mathbf{W}_{i,M}, i=1,2 \quad (14)$$

Where, $\hat{\mathbf{W}}_1$ and $\hat{\mathbf{W}}_2$ are the estimated value, and define estimated error $\tilde{\mathbf{W}}_i = \mathbf{W}_i^* - \hat{\mathbf{W}}_i, i=1,2$.

Hypothesis control input \mathbf{u}_{ns} is bounded, $|\mathbf{u}_{ns}| \leq \mathbf{u}_d$, then $\hat{f}(\hat{\mathbf{x}}) = \hat{\mathbf{W}}_1^T \hat{\mathbf{h}}_1(x)$, $\hat{g}(\hat{\mathbf{x}}) = \hat{\mathbf{W}}_2^T \hat{\mathbf{h}}_2(x)$, the robust term $\nu(t)$ can be express as follows

$$\nu_i(t) = -D_i \frac{\tilde{y}_i}{|\tilde{y}_i|}, \quad i=1,2 \quad (15)$$

Where, $D_1 \geq \beta_1 \sigma_M$, $D_2 \geq \beta_2 \sigma_M \mathbf{u}_d$, $\sigma_M = \sigma_{\max}[L^{-1}(s)]$, $\sigma_{\max}[\cdot]$ is the maximum singular value, $L^{-1}(s)$ is the transfer function of pole stability.

Design adaptive network control law as follows

$$\begin{cases} \dot{\hat{\mathbf{W}}}_1 = F_1 \hat{\mathbf{h}}_1 \tilde{y} - \kappa_1 F_1 |\tilde{y}| \hat{\mathbf{W}}_1 \\ \dot{\hat{\mathbf{W}}}_2 = F_2 \hat{\mathbf{h}}_2 \tilde{y} \mathbf{u}_{ns} - \kappa_2 F_2 |\tilde{y}| \hat{\mathbf{W}}_2 \end{cases} \quad (16)$$

Where, $F_i = F_i^T > 0$, $\kappa_i > 0$, $i=1,2$.

Design sliding mode function as follows

$$s(t) = c\mathbf{e}(t) + \dot{\mathbf{e}}(t) \quad (11)$$

Where, c must meet Hurwitz condition, that is $c > 0$.

Define Lyapunov function

$$V_c = \frac{1}{2} s^2 \quad (12)$$

Eqs.(12) of the time t derivative

$$\dot{s}(t) = c(\dot{\hat{\mathbf{x}}}_1 - \dot{\mathbf{x}}_d) + (\hat{\mathbf{x}}_2 - \ddot{\mathbf{x}}_d) \quad (13)$$

According to the observer Eqs.(11)

$$\dot{\hat{\mathbf{x}}}_1 = \hat{\mathbf{x}}_2 + K_1(\mathbf{x}_1 - \hat{\mathbf{x}}_1) \quad (14)$$

$$\dot{\hat{\mathbf{x}}}_2 = \hat{f}(\hat{\mathbf{x}}) + \hat{g}(\hat{\mathbf{x}})\mathbf{u}_{ns} - \nu(t) + K_2(\mathbf{x}_1 - \hat{\mathbf{x}}_1) \quad (15)$$

Combine Eqs. (13), Eqs. (14) and Eqs. (15)

$$\begin{aligned} \dot{s}(t) = & c((\hat{\mathbf{x}}_2 + K_1(\mathbf{x}_1 - \hat{\mathbf{x}}_1)) - \dot{\mathbf{x}}_d) + \hat{f}(\hat{\mathbf{x}}) \\ & + \hat{g}(\hat{\mathbf{x}})\mathbf{u}_{ns} - \nu(t) + K_2(\mathbf{x}_1 - \hat{\mathbf{x}}_1) - \ddot{\mathbf{x}}_d \end{aligned} \quad (16)$$

In order to guarantee $\dot{V}_c = s\dot{s} < 0$, design sliding mode control law as follows

$$\begin{aligned} \mathbf{u}_{ns} = & \frac{1}{g} (-c(\dot{\mathbf{x}}_2 + K_1(\mathbf{x}_1 - \hat{\mathbf{x}}_1) - \dot{\mathbf{x}}_d) - \hat{\mathbf{f}} \\ & + v(t) - K_2(\mathbf{x}_1 - \hat{\mathbf{x}}_1) + \ddot{\mathbf{x}}_d - \eta \text{sgn}(s)) \end{aligned} \quad (17)$$

Where, $\eta > 0$, then $s\dot{s} = -\eta|s| < 0$, thus $\dot{V}_c \leq 0$. When $s = 0$, $\dot{V}_c = 0$.

4 SIMULATION EXAMPLES

To show the performance of the proposed controller, a simulation is carried out on a planar space manipulator system shown in Fig.1. The actual plant parameters of the system are as follows: $m_0 = 40\text{kg}$, $m_1 = 2\text{kg}$, $m_2 = 1\text{kg}$; $j_0 = 26.67\text{kg} \cdot \text{m}^2$, $j_1 = 2\text{kg} \cdot \text{m}^2$, $j_2 = 1\text{kg} \cdot \text{m}^2$, $J_1 = J_2 = 0.5\text{kg} \cdot \text{m}^2$.

The first and second flexible joint drive motor joint torsional stiffness are $k_1 = 100\text{N} \cdot \text{m}/\text{rad}$ and $k_2 = 100\text{N} \cdot \text{m}/\text{rad}$.

At the same time, the relevant parameters of the control system are as follows: $L^{-1}(s) = \frac{1}{s + 0.5}$, $K_1 = \text{diag}[400 \ 400]$, $K_2 = \text{diag}[800 \ 800]$, $F_1 = \text{diag}[500 \ 500]$, $F_2 = \text{diag}[5 \ 5]$, $c = 20$, $\eta = 0.1$, $\kappa_1 = \kappa_2 = 0.01$.

The desired trajectories of links' joint angles are chosen as $q_{1d} = 7\pi/16$, $q_{2d} = 3\pi/8$, where the unit is in radians.

The initial states of the system are chosen as $q_0(0) = 1.68$, $q_1(0) = 1.25$, $q_2(0) = 1.25$, where the unit is in radians.

Combination the control law of fast subsystem, the control law of slow subsystem (17) and flexible joint compensator, use the proposed control algorithm which is based on adaptive observer sliding mode control in this paper to simulation the free-floating flexible-joint space robot system. The time taking in the simulation is 10 s. Fig.3 is the base attitude trajectory comparison between the open flexible joint compensator and close flexible joint compensator. Fig.4 is the angular displacements q_1 and q_2 comparison between the open flexible joint compensator and close flexible joint compensator.

It can be seen from Fig.3 and Fig.4 that good tracking performance is achieved through the application of the proposed control algorithm. Comparison the simulation results between open and close flexible joint compensation controller, it can be seen the limitation of joint flexibility for applications of the traditional singular perturbation control technique in space robot.

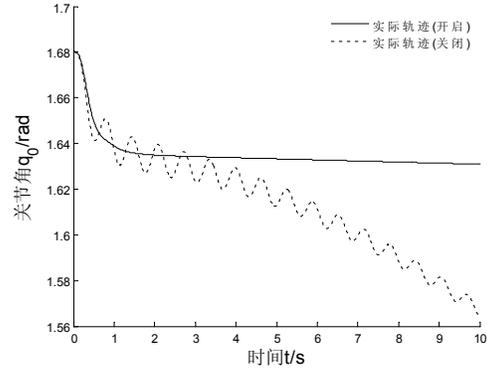


Fig.3 The base attitude trajectory (open and close flexible joint compensator).

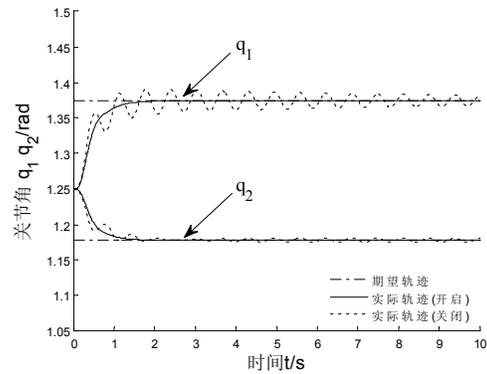


Fig.4 The angular q_1 and q_2 displacements (open and close flexible joint compensator).

5 CONCLUSION

In order to solve the problem of trajectory tracking and vibration suppression control for unknown parameter free-floating space robot system with flexible joints. First, introduced a joint flexibility compensation controller to improve the equivalent stiffness of the joint; then used the singular perturbation method, and the system is decomposed into a slow subsystem which is described joint rigid trajectory tracking and a fast subsystem which is described the vibration of flexible joints. For the slow subsystem, proposed the based on adaptive observer sliding mode control method for flexible-joints space robot, designed an observer to monitor the state variables of flexible-joints space robot system, and applies sliding mode variable structure to design the controller. The fast subsystem used the speed difference between the feedback control law to suppress the elastic vibration of flexible joints which is caused by system, guarantee the stability of the system. Numerical simulation results show that the proposed integrated control algorithm can precision and stability control the joint tracking of the space robot with flexible joints, and active vibration suppression of flexible joints; the reliability and validity of the controller are proved.

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References

- [1] Kumar A, Pathak P, Sukavanam N (2011) Reduced model based control of two link flexible space robot. *Intelligent Control and Automation* 2(2):112-120.
- [2] Sabatini M, Gasbarri P, Monti R, Palmerini G B (2012) Vibration control of a flexible space manipulator during on orbit operations. *Acta Astronautica* 73(2):109-121.
- [3] Spong M W (1987) Modeling and control of elastic joint robots. *Journal of Dynamics Systems, Measurement, and Control* 109(4):310-319.
- [4] Nanas K (2011) On the use of free-floating space robots in the presence of angular momentum. *Intelligent Service Robotics* 4(1):3-15.
- [5] Lightcap C A, Banks S A (2010) An extended Kalman filter for real-time estimation and control of a rigid-link flexible joint manipulator. *IEEE Transactions on Control Systems Technology* 18(1):91-103.
- [6] Park Y J, Wan K C (2013) External torque-sensing algorithm for flexible-joint robot based on Kal-man filter. *Electronics Letters* 49(14):877-878.
- [7] Vakil M, Fotouhi R, Nikiforuk P N (2012) A new method for dynamic modeling of flexible-link flexible-joint manipulators. *Journal of Vibration and Acoustics* 134(1): 14503-14513.
- [8] Yeon J S, Yim J, Park H (2011) Robust control using recursive design method for flexible joint robot manipulator. *Journal of Mechanical Science and Technology* 25(12):3205-3213.
- [9] Bechlioulis, C P, and Rovithakis, G A (2008) Robust adaptive control of feedback linearizable mimo nonlinear systems with prescribed performance. *IEEE Trans. Autom. Control* 53(9):2090-2099.
- [10] Koofigar H R (2014) Adaptive tracking with external force disturbance rejection for uncertain robotic systems. *International Journal of Control Automation and Systems* 12(1):169-176.
- [11] Chen Zhiyong, Chen Li (2011) Study on Dynamics modeling and singular perturbation control of free-floating space robot with flexible joints. *China Mechanical Engineering* 18(22):2151-2155.
- [12] Aghili F (2009) Coordination control of a free-flying manipulator and its base attitude to capture and detumble a noncooperative satellite. In: *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*,

St.Louis,USA,2009,pp.2365-2372.

- [13] Liu Y C, Jin M H, Liu H (2008) Singular perturbation control for flexible-joint manipulator based on flexibility compensation. *Robot* 30(5) 460-466.