Fault-tolerant Control And Active Vibration Suppression of Free-floating Flexible Space Robot

*Ziwang Zhao[1], Li Chen[2],

[1]Institute of Mechanical Engineering and Automation, Fuzhou University, Institute of Mechanical Engineering and Automation, Fuzhou University, 2 Xue Yuan Road, University Town, Fuzhou, Fujian, 350116, China, E-mail: zzwbaymax@126.com

[2]Institute of Mechanical Engineering and Automation, Fuzhou University, Institute of Mechanical Engineering and Automation, Fuzhou University, 2 Xue Yuan Road, University Town, Fuzhou, Fujian, 350116, china, E-mail: chnle@fzu.edu.cn

ouABSTRACT

A method of decentralized fault tolerant control, based on decentralized back-stepping terminal sliding mode neural network control, is proposed for flexible space robot in the presence of partial loss of actuator effectiveness, where base's attitude is controlled and base's position isn't controlled. Based on Lagrange second dynamics equation, assumed mode method and linear momentum conservation of the system, the dynamic model of flexible space robot is established. Furthermore, the system is decentralized into some subsystems, and then an effective factor is integrated into the dynamic model of each subsystem. To solve the problem of trajectory tracking control and active vibration suppression of the flexible space robot, the singular perturbation approach is adopted to decompose the robot system into a slow subsystem and a fast subsystem so that controller can be designed separately for the subsystems. With regard to the slow subsystem, the back-stepping terminal sliding mode techniques is used to design the control scheme which guarantees that the tracking errors of base's attitude angle and space manipulator's joint angles converge to zero in finite time, and with the help of radial basis function (RBF) neural networks, uncertain terms and unknown terms are estimated with adaptive compensation of estimation error. As for the fast subsystem, PD control is used to weaken the chattering of the terminal sliding mode controller and the vibration of the flexible arm effectively. Finally, the simulation results prove the validity of the controller.

1 INTRODUCTION

With the exploration of the universe, such as the launch of the satellite and the establishment of the space station, space robot will be more and more applied to high precision task and high stability task [1-5].Space robot can be used for installation and replacement of space station components, maintenance and recovery of failure satellites, so reliability and safety of space robot is very important. The reliability of the system depends largely on the operation of actuators and sensors. Because space robot needs to work in the harsh environment of high vacuum, weightlessness, ultra low temperature and strong radiation for a long time, actuators or sensors failure is inevitable that has a bad effect on reliability and stability of the system. So the research on fault tolerant control is very practical [6-8].

Furthermore, as the result of lightweight design of space manipulator, the flexible issues of space robot need to be taken into consideration. The vibration of flexible arm has a bad influence on the precision of control and stability of operation, so the active suppression of vibration is essential for the system [9-10].

In this paper, based on singular perturbation theory, a composite control of the slow subsystem and the fast subsystem control scheme is proposed. A planar flexible arm space robot system is simulated to verify the proposed control scheme.

2 DYNAMICS OF THE SYSTEM

Without any loss of generality, a planar flexible-arm space robot system is considered here, Fig.1. The system consists of the rigid base B_0 , the rigid link B_1 , and the flexible link B_2 . O_0 coincides with the mass center of O_{C0} of B_0 , O_i $(i=1,2)$ is the rotational center of the revolute joint between B_{i-1} and B_i . O_{Ci} $(i=1,2)$ is the mass center of B_i . x_i is the symmetrical axis of each link.

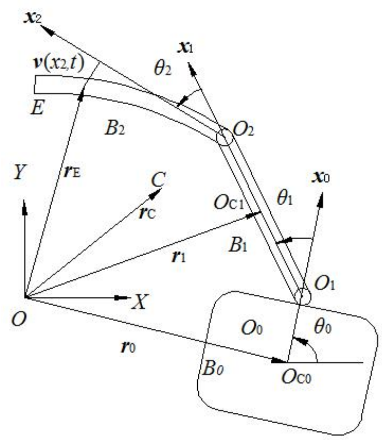

Fig.1. A planar flexible-arm space robot system

With the second Lagrange Equation, assumed mode method and momentum conservation law, the dynamic equation of flexible-arm space robot can be derived as follow

$$\boldsymbol{D}(\boldsymbol{\theta},\boldsymbol{\delta})\begin{bmatrix}\ddot{\boldsymbol{\theta}}\\ \ddot{\boldsymbol{\delta}}\end{bmatrix}+\boldsymbol{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\delta},\dot{\boldsymbol{\delta}})\begin{bmatrix}\dot{\boldsymbol{\theta}}\\ \dot{\boldsymbol{\delta}}\end{bmatrix}+\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{K\delta}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\tau}\\ \boldsymbol{0}\end{bmatrix} \quad (1)$$

where $\boldsymbol{\theta}=[\theta_0,\theta_1,\theta_2]^{\mathrm{T}}\in R^{3\times1}$ is generalized coordinate vector of the system; $\boldsymbol{\delta}=[\delta_1,\delta_2]^{\mathrm{T}}\in R^{2\times1}$ is the vector of mode; $\boldsymbol{D}(\boldsymbol{\theta},\boldsymbol{\delta})\in R^{5\times5}$ is symmetric positive-definite inertial matrix; $\boldsymbol{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\delta},\dot{\boldsymbol{\delta}})[\dot{\boldsymbol{\theta}};\dot{\boldsymbol{\delta}}]\in R^{5\times1}$ is the vector that consists of centripetal and Coriolis forces; $\boldsymbol{K}=diag(k_1,k_2)\in R^{2\times2}$ is the flexible link's stiffness matrix; $\boldsymbol{\tau}=[\tau_0,\tau_1,\tau_2]^{\mathrm{T}}$ is the vector of input torques.

In the process of operation, the deformation and active vibration of flexible-arm space robot can't be ignored. In order to realize the active suppression of flexible vibration and accurate tracking of the desired trajectory, the system can be decomposed into a slow subsystem and a fast subsystem based on singular perturbation theory. The slow subsystem's control torque is τ_s which is used to realize accurate tracking of the desired trajectory. The fast subsystem's control torque is τ_f which is used for achieving the active suppression of flexible vibration, and $\boldsymbol{\tau}=\boldsymbol{\tau}_s+\boldsymbol{\tau}_f$.

In order to perform the singular perturbation decomposition, equation (1) can be written as

$$\begin{bmatrix}\boldsymbol{D}_{rr} & \boldsymbol{D}_{r\delta}\\ \boldsymbol{D}_{\delta r} & \boldsymbol{D}_{\delta\delta}\end{bmatrix}\begin{bmatrix}\ddot{\boldsymbol{\theta}}\\ \ddot{\boldsymbol{\delta}}\end{bmatrix}+\begin{bmatrix}\boldsymbol{H}_{rr} & \boldsymbol{H}_{r\delta}\\ \boldsymbol{H}_{\delta r} & \boldsymbol{H}_{\delta\delta}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}\\ \dot{\boldsymbol{\delta}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\tau}\\ \boldsymbol{0}\end{bmatrix} \quad (2)$$

where $\boldsymbol{D}_{rr}\in R^{3\times3}$, $\boldsymbol{D}_{r\delta}\in R^{3\times2}$, $\boldsymbol{D}_{\delta r}\in R^{2\times3}$ and $\boldsymbol{D}_{\delta\delta}\in R^{2\times2}$ are the sub-matrices of $\boldsymbol{D}$; $\boldsymbol{H}_{rr}\in R^{3\times3}$, $\boldsymbol{H}_{r\delta}\in R^{3\times2}$, $\boldsymbol{H}_{\delta r}\in R^{2\times3}$ and $\boldsymbol{H}_{\delta\delta}\in R^{2\times2}$ are the sub-matrices of $\boldsymbol{H}$.

Since the inertia matrix $\boldsymbol{D}$ in equation (1) is positive definite, it can be inverted and denoted by $\boldsymbol{D}^{-1}$, which can be partitioned.

$$\boldsymbol{D}^{-1}=\begin{bmatrix}\boldsymbol{D}_{rr} & \boldsymbol{D}_{r\delta}\\ \boldsymbol{D}_{\delta r} & \boldsymbol{D}_{\delta\delta}\end{bmatrix}^{-1}=\begin{bmatrix}\boldsymbol{M}_{rr} & \boldsymbol{M}_{r\delta}\\ \boldsymbol{M}_{\delta r} & \boldsymbol{M}_{\delta\delta}\end{bmatrix} \quad (3)$$

where

$M_{rr}=(\boldsymbol{D}_{rr}-\boldsymbol{D}_{r\delta}\boldsymbol{D}_{\delta\delta}^{-1}\boldsymbol{D}_{\delta r})^{-1}$,

$\boldsymbol{M}_{r\delta}=-\boldsymbol{D}_{rr}^{-1}\boldsymbol{D}_{r\delta}(\boldsymbol{D}_{\delta\delta}-\boldsymbol{D}_{\delta r}\boldsymbol{D}_{rr}^{-1}\boldsymbol{D}_{r\delta})^{-1}$,

$\boldsymbol{M}_{\delta r}=-\boldsymbol{D}_{\delta\delta}^{-1}\boldsymbol{D}_{\delta r}(\boldsymbol{D}_{rr}-\boldsymbol{D}_{r\delta}\boldsymbol{D}_{\delta\delta}^{-1}\boldsymbol{D}_{\delta r})^{-1}$,

$\boldsymbol{M}_{\delta\delta}=(\boldsymbol{D}_{\delta\delta}-\boldsymbol{D}_{\delta r}\boldsymbol{D}_{rr}^{-1}\boldsymbol{D}_{r\delta})^{-1}$.

Hence, $\ddot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\delta}}$ can be written as follows

$$\ddot{\boldsymbol{\theta}} = -\boldsymbol{M}_{rr}(\boldsymbol{H}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{r\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{r\delta}(\boldsymbol{H}_{\delta r}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{\delta\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{r\delta}\boldsymbol{K}\boldsymbol{\delta} + \boldsymbol{M}_{rr}\boldsymbol{\tau} \tag{4}$$

$$\ddot{\boldsymbol{\delta}} = -\boldsymbol{M}_{\delta r}(\boldsymbol{H}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{r\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{\delta\delta}(\boldsymbol{H}_{\delta r}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{\delta\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{\delta\delta}\boldsymbol{K}\boldsymbol{\delta} + \boldsymbol{M}_{\delta r}\boldsymbol{\tau} \tag{5}$$

$u^2 = 1/\min\{k_1, k_2\}$ is defined as the singular perturbation factor. $\boldsymbol{z} = \boldsymbol{\delta}/u^2$ and $\boldsymbol{K}_s = u^2\boldsymbol{K}$ are substituted into equation (4) and (5), then equation (6) and (7) are obtained as follows

$$\ddot{\boldsymbol{\theta}} = -\boldsymbol{M}_{rr}(\boldsymbol{H}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{r\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{r\delta}(\boldsymbol{H}_{\delta r}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{\delta\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{r\delta}\boldsymbol{K}_s\boldsymbol{z} + \boldsymbol{M}_{rr}\boldsymbol{\tau} \tag{6}$$

$$u^2\ddot{\boldsymbol{z}} = -\boldsymbol{M}_{\delta r}(\boldsymbol{H}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{r\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{\delta\delta}(\boldsymbol{H}_{\delta r}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{\delta\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{\delta\delta}\boldsymbol{K}_s\boldsymbol{z} + \boldsymbol{M}_{\delta r}\boldsymbol{\tau} \tag{7}$$

If $u = 0$, the slow subsystem's dynamics equation can be derived as

$$\overline{\boldsymbol{D}}_{rr}\ddot{\boldsymbol{\theta}} + \overline{\boldsymbol{H}}_{rr}\dot{\boldsymbol{\theta}} = \boldsymbol{\tau}_s \tag{8}$$

In addition, slowly varying manifold expression can be derived as

$$\bar{\boldsymbol{z}} = \boldsymbol{K}_s^{-1}\overline{\boldsymbol{M}}_{\delta\delta}^{-1}(-\overline{\boldsymbol{M}}_{\delta r}\overline{\boldsymbol{H}}_{rr}\dot{\boldsymbol{\theta}} - \overline{\boldsymbol{M}}_{\delta\delta}\overline{\boldsymbol{H}}_{\delta r}\dot{\boldsymbol{\theta}} + \overline{\boldsymbol{M}}_{\delta r}\boldsymbol{\tau}) \tag{9}$$

To derived the fast subsystem's dynamics equation, a fast time scale $t_f = t/u$ and boundary layer correction terms $\boldsymbol{z}_{f1} = \boldsymbol{z} - \bar{\boldsymbol{z}}$ and $\boldsymbol{z}_{f2} = u\dot{\boldsymbol{z}}$ are introduced.

Because of $d\bar{\boldsymbol{z}}/dt_f = u\bar{\boldsymbol{z}} = 0$, the boundary layer system can be expressed as

$$\frac{d\boldsymbol{z}_{f1}}{dt_f} = \boldsymbol{z}_{f2} \tag{10}$$

$$\frac{d\boldsymbol{z}_{f2}}{dt_f} = -\boldsymbol{M}_{\delta r}(\boldsymbol{H}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{r\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{\delta\delta}(\boldsymbol{H}_{\delta r}\dot{\boldsymbol{\theta}} + \boldsymbol{H}_{\delta\delta}\dot{\boldsymbol{\delta}}) - \boldsymbol{M}_{\delta\delta}\boldsymbol{K}_s(\boldsymbol{z}_{f1} + \bar{\boldsymbol{z}}) + \boldsymbol{M}_{\delta r}(\boldsymbol{\tau}_s + \boldsymbol{\tau}_f) \tag{11}$$

When $u = 0$, the fast subsystem's dynamics equations are derived as

$$\frac{d\boldsymbol{z}_{f1}}{dt_f} = \boldsymbol{z}_{f2} \tag{12}$$

$$\frac{d\boldsymbol{z}_{f2}}{dt_f} = -\overline{\boldsymbol{M}}_{\delta\delta}\boldsymbol{K}_s\boldsymbol{z}_{f1} + \overline{\boldsymbol{M}}_{\delta r}\boldsymbol{\tau}_f \tag{13}$$

The fast subsystem state equation of flexible-arm space robot is written as

$$\frac{d\boldsymbol{Z}_f}{dt_f} = \boldsymbol{A}_f\boldsymbol{Z}_f + \boldsymbol{B}_f\boldsymbol{\tau}_f \tag{14}$$

Where $\boldsymbol{Z}_f = [z_{f1} \ \ z_{f2}]^{\mathrm{T}}$, $\boldsymbol{A}_f = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\overline{\boldsymbol{M}}_{\delta\delta}\boldsymbol{K}_s & \boldsymbol{0} \end{bmatrix}$,

$\boldsymbol{B}_f = [\boldsymbol{0} \ \ \overline{\boldsymbol{M}}_{\delta r}]^{\mathrm{T}}$.

In short, the singular perturbation mode of flexible-arm space robot is made up of equation (8) and (14).

3 Slow subsystem decentralized fault tolerant controller design

3.1 Slow subsystem decentralized dynamic model

In order to achieve individual control for each subsystem, slow subsystem is decentralized into three subsystems. Based on the equation (8), the subsystems' dynamic models of slow subsystem are established as follows

$$\overline{D}_{rri}(\theta_i)\ddot{\theta}_i + \overline{H}_{rri}(\theta_i,\dot{\theta}_i)\dot{\theta}_i + Z_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) = \tau_{si} \quad (i = 1,2,3) \tag{15}$$

$$Z_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) = \left\{ \sum_{j=1, j\neq i}^{3} \overline{D}_{rrij}(\boldsymbol{\theta})\ddot{\theta}_j + [\overline{D}_{rrii}(\boldsymbol{\theta}) - \overline{D}_{rri}(\theta_i)]\ddot{\theta}_i \right\} + \left\{ \sum_{j=1, j=i}^{3} \overline{H}_{rrij}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\theta}_j + [\overline{H}_{rrii}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) - \overline{H}_{rri}(\theta_i,\dot{\theta}_i)]\dot{\theta}_i \right\} \tag{16}$$

Where θ_i, $\dot{\theta}_i$, $\ddot{\theta}_i$ and τ_{si} are the ith element of the vectors $\boldsymbol{\theta}$, $\dot{\boldsymbol{\theta}}$, $\ddot{\boldsymbol{\theta}}$ and $\boldsymbol{\tau}_s$; $\overline{D}_{rrij}$ and $\overline{H}_{rrij}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ is the ijth element of the matrix $\overline{\boldsymbol{D}}_{rr}(\boldsymbol{\theta})$ and $\overline{\boldsymbol{H}}_{rr}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ respectively.

For the subsystem dynamic model shown as (15), the dynamic model with actuator's partial loss is described as

$$\overline{D}_{rri}(\theta_i)\ddot{\theta}_i + \overline{H}_{rri}(\theta_i,\dot{\theta}_i)\dot{\theta}_i + Z_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) = \rho_i\tau_{si} \quad (i=1,2,3) \tag{17}$$

where ρ_i is the effective factor of ith actuator, $0<\rho_i\le 1$ $(i=1,2,3)$, and it stands for the degree of actuator's fault.

Assuming $\boldsymbol{x}_i = [x_{i1}\ \ x_{i2}]^{\mathrm{T}} = [\theta_i\ \ \dot{\theta}_i]^{\mathrm{T}}$ $(i=1,2,3)$, equation (17) can be written as

$$S_i\begin{cases}\dot{x}_{i1} = x_{i2}\\ \dot{x}_{i2} = f_i(\theta_i,\dot{\theta}_i) + g_i(\theta_i)\tau_{si} + h_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}})\\ y_i = x_{i1}\end{cases} \tag{18}$$

where $\boldsymbol{x}_i$ is the state variable of subsystem S_i, y_i is the output variable of subsystem S_i, and $f_i(\theta_i,\dot{\theta}_i)$, $g_i(\theta_i)$ and $h_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}})$ can be expressed as

$$f_i(\theta_i,\dot{\theta}_i) = -\overline{D}_{rri}^{-1}(\theta_i)\overline{H}_{rri}(\theta_i,\theta_i) \tag{19}$$

$$g_i(\theta_i) = \rho_i\overline{D}_{rri}^{-1} \tag{20}$$

$$h_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) = -\overline{D}_{rri}(\theta_i)Z_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) \tag{21}$$

3.2 Slow subsystem decentralized back-stepping terminal sliding mode fault tolerant controller design

Define the position tracking error e_{i1} and velocity tracking error e_{i2}

$$\begin{cases}e_{i1} = \theta_i - \theta_{id}\\ e_{i2} = \dot{\theta}_i - \alpha_i\end{cases} \tag{22}$$

where α_i is virtual control variable, it is defined as

$$\alpha_i = \dot{\theta}_{id} - c_i e_{i1} \tag{23}$$

where $c_i > 0$.

Differentiating equation (22) with respect to time t yields

$$\begin{cases}\dot{e}_{i1} = \dot{\theta}_i - \dot{\theta}_{id}\\ \dot{e}_{i2} = f_i(\theta_i,\dot{\theta}_i) + g_i(\theta_i)\tau_{si} + h(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) - \dot{\alpha}_i\end{cases} \tag{24}$$

As to the first phase of tracking error, select the Lyapunov function candidate

$$V_{i1} = \frac{1}{2}e_{i1}^2 \tag{25}$$

Differentiating equation (25) with respect to time t yields

$$\dot{V}_{i1} = e_{i1}\dot{e}_{i1} = e_{i1}\dot{e}_{i2} - c_i e_{i1}^2 \tag{26}$$

The terminal sliding mode is introduced for the tracking error of subsystem

$$s_i = e_{i2} + \beta_i e_{i1}^{\delta_i/\lambda_i} \tag{27}$$

where λ_i and δ_i are positive odd numbers, $1<\delta_i/\lambda_i<2$, and β_i is positive constant.

Differentiating equation (27) with respect to time t yields

$$\begin{aligned}\dot{s}_i &= \dot{e}_{i2} + \beta_i\frac{\delta_i}{\lambda_i}e_{i1}^{\delta_i/\lambda_i-1}\dot{e}_{i1}\\ &= f_i(\theta_i,\dot{\theta}_i) + g_i(\theta_i)\tau_{si} + h_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) - \dot{\alpha}_i + \beta_i\frac{\delta_i}{\lambda_i}e_{i1}^{\delta_i/\lambda_i-1}\dot{e}_{i1}\end{aligned} \tag{28}$$

The unknown and uncertain terms of system dynamics model are estimated by using radial basis function neural networks.

$$f_i(\theta_i,\dot{\theta}_i,\boldsymbol{W}_{if}) = \boldsymbol{W}_{if}^{\mathrm{T}}\boldsymbol{\Phi}_{if}(\theta_i,\dot{\theta}_i) + \varepsilon_{if} \quad \|\varepsilon_{if}\| \le \varepsilon_{i1} \tag{29}$$

$$g_i(\theta_i,\boldsymbol{W}_{ig}) = \boldsymbol{W}_{ig}^{\mathrm{T}}\boldsymbol{\Phi}_{ig}(\theta_i) + \varepsilon_{ig} \quad \|\varepsilon_{ig}\| \le \varepsilon_{i2} \tag{30}$$

where $\boldsymbol{W}_{if}$ and $\boldsymbol{W}_{ig}$ is the RBF neural networks' ideal weights of unknown term $f_i(\theta_i,\dot{\theta}_i)$ and uncertain term $g_i(\theta_i)$ respectively, and $\boldsymbol{\Phi}(\cdot)$ is ideal basis function of RBF neural networks, and ε_{if} and ε_{ig} are the estimation error of RBF neural networks, and ε_{i1} and ε_{i2} are the known positive numbers.

$\hat{\boldsymbol{W}}_{if}$ and $\hat{\boldsymbol{W}}_{ig}$ is defined as the estimation of $\boldsymbol{W}_{if}$ and

$\boldsymbol{W}_{ig}$ respectively; $\hat{\boldsymbol{\Phi}}_{if}$ and $\hat{\boldsymbol{\Phi}}_{ig}$ is defined as the estimation of $\boldsymbol{\Phi}_{if}$ and $\boldsymbol{\Phi}_{ig}$ respectively; $\hat{f}(\theta_i,\dot{\theta}_i,\hat{\boldsymbol{W}}_{if})$ and $\hat{g}(\theta_i,\hat{\boldsymbol{W}}_{ig})$ is defined as the estimation of $f_i(\theta_i,\dot{\theta}_i)$ and $g_i(\theta_i)$ respectively.

Then

$$\hat{f}_i(\theta_i,\dot{\theta}_i,\hat{\boldsymbol{W}}_{if})=\hat{\boldsymbol{W}}_{if}^{\mathrm{T}}(\theta_i,\dot{\theta}_i)\hat{\boldsymbol{\Phi}}(\theta_i,\dot{\theta}_i) \tag{31}$$

$$g_i(\theta_i,\hat{\boldsymbol{W}}_{ig})=\hat{\boldsymbol{W}}_{ig}^{\mathrm{T}}\hat{\boldsymbol{\Phi}}_{ig}(\theta_i) \tag{32}$$

Define estimation errors

$\widetilde{\boldsymbol{W}}_{if}=\boldsymbol{W}_{if}-\hat{\boldsymbol{W}}_{if}$, $\widetilde{\boldsymbol{W}}_{ig}=\boldsymbol{W}_{ig}-\hat{\boldsymbol{W}}_{ig}$, $\widetilde{\boldsymbol{\Phi}}_{if}=\boldsymbol{\Phi}_{if}-\hat{\boldsymbol{\Phi}}_{if}$, $\widetilde{\boldsymbol{\Phi}}_{ig}=\boldsymbol{\Phi}_{ig}-\hat{\boldsymbol{\Phi}}_{ig}$.

$$f_i(\theta_i,\dot{\theta}_i,\boldsymbol{W}_{if})-\hat{f}_i(\theta_i,\dot{\theta}_i,\hat{\boldsymbol{W}}_{if})=\widetilde{\boldsymbol{W}}_{if}^{\mathrm{T}}\hat{\boldsymbol{\Phi}}_{if}+\boldsymbol{W}_{if}^{\mathrm{T}}\widetilde{\boldsymbol{\Phi}}_{if}+\varepsilon_{if} \tag{33}$$

$$g_i(\theta_i,\boldsymbol{W}_{ig})-\hat{g}_i(\theta_i,\hat{\boldsymbol{W}}_{ig})=\widetilde{\boldsymbol{W}}_{ig}^{\mathrm{T}}\hat{\boldsymbol{\Phi}}_{ig}+\boldsymbol{W}_{ig}^{\mathrm{T}}\widetilde{\boldsymbol{\Phi}}_{ig}+\varepsilon_{ig} \tag{34}$$

The interconnection term $Z_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}})$ is explicitly bounded by

$$\left|h_i(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}})\right|\le\sum_{j=1}^{n}d_{ij}S_j \tag{35}$$

where $d_{ij}\ge 0$, $S_j=1+\left|s_j\right|+\left|s_j\right|^2$.

The interconnection term is compensated by RBF neural networks, and the compensation term is defined as $p_i(\left|s_j\right|)=n\max_{ij}\{d_{ij}\}S_i$, and its estimation can be expressed as

$$\hat{p}(\left|s_i\right|,\hat{\boldsymbol{W}}_{ip})=\hat{\boldsymbol{W}}_{ip}^{\mathrm{T}}\hat{\boldsymbol{\Phi}}_{ip}(\left|s_i\right|) \tag{36}$$

where $\hat{\boldsymbol{W}}_{ip}^{\mathrm{T}}$ is the estimation of $\boldsymbol{W}_{ip}^{\mathrm{T}}$, $\widetilde{\boldsymbol{W}}_{ip}=\boldsymbol{W}_{ip}^{\mathrm{T}}-\hat{\boldsymbol{W}}_{ip}^{\mathrm{T}}$.

The estimation errors are defined as follows

$$w_{i1}=\boldsymbol{W}_{if}^{\mathrm{T}}\widetilde{\boldsymbol{\Phi}}_{if}+\boldsymbol{W}_{ig}^{\mathrm{T}}\widetilde{\boldsymbol{\Phi}}_{ig}\tau_{si}+\varepsilon_{if}+\varepsilon_{ig}\tau_{si} \tag{37}$$

$$w_{i2}=p_i(\left|s_i\right|)-\boldsymbol{W}_{ip}^{\mathrm{T}}\hat{\boldsymbol{\Phi}}_{ip}(\left|s_i\right|) \tag{38}$$

$$w_i=\left|w_{i1}\right|+\left|w_{i2}\right| \tag{39}$$

where w_i is bounded by $\left|w_i\right|\le\eta_i$, $\eta_i\ge 0$.

The terminal factor is introduced for the system's velocity tracking error.

$$\dot{s}_i=-\varphi_i s_i-r_i s_i^{\frac{m_i}{n_i}} \tag{40}$$

Where φ_i and r_i are positive constants, $0<\dfrac{m_i}{n_i}<1$.

Therefore the decentralized fault-tolerant controller of subsystem is designed as

$$\begin{aligned}\tau_{si}=-[&\hat{f}_i(\theta_i,\dot{\theta}_i,\hat{\boldsymbol{W}}_{if})+\mathrm{sgn}(s_i)\hat{p}_i(\left|s_i\right|,\hat{\boldsymbol{W}}_{ip})-\dot{\alpha}_i+\\&\beta_i\frac{\delta_i}{\lambda_i}e_{i1}^{\frac{\delta_i}{\lambda_i}-1}\dot{e}_{i1}+e_{i1}+\mathrm{sgn}(s_i)\hat{\eta}_i+\\&\varphi_i s_i+r_i s_i^{\frac{m_i}{n_i}}]/\hat{g}(\theta_i,\hat{W}_{ig})\end{aligned} \tag{41}$$

where $\hat{\boldsymbol{W}}_{if}$, $\hat{\boldsymbol{W}}_{ig}$, $\hat{\boldsymbol{W}}_{ip}$ and $\hat{\boldsymbol{\eta}}$ are adjusted adaptively by

$$\dot{\hat{\boldsymbol{W}}}_{if}=\boldsymbol{\Theta}_{if}s_i\hat{\boldsymbol{\Phi}}_{if}(\theta_i,\dot{\theta}_i) \tag{42}$$

$$\dot{\hat{\boldsymbol{W}}}_{ig}=\boldsymbol{\Theta}_{ig}s_i\hat{\boldsymbol{\Phi}}_{ig}(\theta_i)\tau_{si} \tag{43}$$

$$\dot{\hat{\boldsymbol{W}}}_{ip}=\boldsymbol{\Theta}_{ip}\left|s_i\right|\hat{\boldsymbol{\Phi}}_{ip}(\left|s_i\right|) \tag{44}$$

$$\dot{\hat{\boldsymbol{\eta}}}=\boldsymbol{\Theta}_{i\eta}\left|s_i\right| \tag{45}$$

3.3 Stability analysis of the low subsystem

As to the second phase of tracking error, select the Lyapunov function candidate

$$\begin{aligned}V_{i2}=&\frac{1}{2}s_i^2+\frac{1}{2}\widetilde{\boldsymbol{W}}_{if}^{\mathrm{T}}\boldsymbol{\Theta}_{if}^{-1}\widetilde{\boldsymbol{W}}_{if}+\frac{1}{2}\widetilde{\boldsymbol{W}}_{ig}^{\mathrm{T}}\boldsymbol{\Theta}_{ig}^{-1}\widetilde{\boldsymbol{W}}_{ig}+\\&\frac{1}{2}\widetilde{\boldsymbol{W}}_{ip}^{\mathrm{T}}\boldsymbol{\Theta}_{ip}^{-1}\widetilde{\boldsymbol{W}}_{ip}+\frac{1}{2}\widetilde{\eta}_i^{\mathrm{T}}\boldsymbol{\Theta}_{i\eta}^{-1}\widetilde{\eta}_i\end{aligned} \tag{46}$$

The positive definite function V is defined as Lyapunov function.

$$V=\sum_{i=1}^{n}(V_{i1}+V_{i2}) \tag{47}$$

$$V=\sum_{i=1}^{n}(\frac{1}{2}e_{i1}^2+\frac{1}{2}s_i^2+\frac{1}{2}\widetilde{\boldsymbol{W}}_{if}^{\mathrm{T}}\boldsymbol{\Theta}_{if}^{-1}\widetilde{\boldsymbol{W}}_{if}+\frac{1}{2}\widetilde{\boldsymbol{W}}_{ig}^{\mathrm{T}}\boldsymbol{\Theta}_{ig}^{-1}\widetilde{\boldsymbol{W}}_{ig}+\frac{1}{2}\widetilde{\boldsymbol{W}}_{ip}^{\mathrm{T}}\boldsymbol{\Theta}_{ip}^{-1}\widetilde{\boldsymbol{W}}_{ip}+\frac{1}{2}\widetilde{\eta}_i^{\mathrm{T}}\boldsymbol{\Theta}_{i\eta}^{-1}\widetilde{\eta}_i) \tag{48}$$

Differentiating equation (47) with respect to time t yields

$$\dot{V}=\sum_{i=1}^{n}(e_{i1}e_{i2}-c_ie_{i1}^2+s_i\dot{s}_i-\widetilde{\boldsymbol{W}}_{if}^{\mathrm{T}}\boldsymbol{\Theta}_{if}^{-1}\dot{\hat{\boldsymbol{W}}}_{if}-\widetilde{\boldsymbol{W}}_{ig}^{\mathrm{T}}\boldsymbol{\Theta}_{ig}^{-1}\dot{\hat{\boldsymbol{W}}}_{ig}-\widetilde{\boldsymbol{W}}_{ip}^{\mathrm{T}}\boldsymbol{\Theta}_{ip}^{-1}\dot{\hat{\boldsymbol{W}}}_{ip}-\widetilde{\boldsymbol{\eta}}_i^{\mathrm{T}}\boldsymbol{\Theta}_{i\eta}^{-1}\dot{\hat{\eta}}_i)$$
$$\le\sum_{i=1}^{n}(-c_ie_{i1}^2-\beta_ie_i^{\frac{\delta_i}{\lambda_i}+1}-\varphi_is_i^2-r_is_i^{\frac{m_i}{n_i}+1}+|s_i||w_{i1}|-|s_i|\boldsymbol{W}_{ip}^{\mathrm{T}}\hat{\boldsymbol{\Phi}}_{ip}-|s_i|\boldsymbol{\eta}_i)+\max_{ij}d_{ij}\sum_{i=1}^{n}|s_i|\sum_{j=1}^{n}S_j \tag{49}$$

If $|s_i|\le|s_j|$, $|S_i|\le|S_j|$. Judging by Chebyshv inequality, then

$$\sum_{i=1}^{n}|s_i|\sum_{j=1}^{n}S_j\le n\sum_{i=1}^{n}|s_i|S_i \tag{50}$$

Substituting (41) and (50) into (49), one can obtain that

$$\dot{V}\le\sum_{i=1}^{n}(-c_ie_{i1}^2-\beta_ie_{i1}^{\frac{\delta_i}{\lambda_i}+1}-\varphi_is_i^2-r_is_i^{\frac{m_i}{n_i}+1})\le 0 \tag{51}$$

This shows that the slow subsystem is stable.

$$V_a=\sum_{i=1}^{n}(c_ie_{i1}^2+\beta_ie_{i1}^{\frac{\delta_i}{\lambda_i}+1}+\varphi_is_i^2+r_is_i^{\frac{m_i}{n_i}+1}) \tag{52}$$

$$\int_0^t V_a dt\le-\int_0^t \dot{V}dt=V(0)-V(t) \tag{53}$$

Because $V(0)$ is bounded and $V(t)$ is not increasing and bounded from below, one can obtain that

$$\lim_{t\to\infty}\int_0^t V_a dt<\infty \tag{54}$$

According to *Barbalat* lemma, it implies that $V_a(t)\to 0$ as $t\to\infty$, which indicates $e_{i1}\to 0$ and $s_i\to 0$ as $t\to\infty$. So the tracking error converges to zero.

4 Fast subsystem controller design-PD feedback control

Equation (14), state equation of fast subsystem, shows that fast subsystem is a linear function of the state variables, so the negative feedback control of state variable can be taken into consideration. The controller of fast subsystem is designed as

$$\tau_f=-K_fZ_f \tag{55}$$

where $K_f\in R^{3\times 4}$.

In summary, a composite control scheme of decentralized terminal sliding mode fault tolerant control and PD feedback control is designed to realize fault tolerant control, precise trajectory tracking and flexible vibration suppression.

5 Numerical Simulation

A planar flexible-arm space robot system shown in Fig.1 is used as an example to illustrate the proposed control schemes. The plant parameters of the system are listed as follows

$m_0=40kg$, $l_0=1.5m$, $J_0=34.17kg\cdot m^2$;

$m_1=2kg$, $l_1=3m$, $d_1=1.5m$, $J_1=3kg\cdot m^2$;

$\rho=1kg/m$, $l_2=3m$, $EI=100N\cdot m^2$.

The desired motions of attitude angle of base and joint angles are chosen as

$\theta_{0d}=0.1\text{rad}$, $\theta_{1d}=0.5\text{rad}$, $\theta_{2d}=0.5\text{rad}$

The initial states of the system are given as

$\theta_0=0.5\text{rad}$, $\theta_1=0\text{rad}$, $\theta_2=0.95\text{rad}$, $\delta_1=0$, $\delta_2=0$.

The effective factors are chosen as

$$\begin{cases}\rho_1(t)=0.8, & t\ge 2\text{s}\\ \rho_2(t)=0.5, & t\ge 5\text{s}\\ \rho_3(t)=0.5, & t\ge 8\text{s}\end{cases} \tag{56}$$

The time taken for simulation is 20 seconds.

As the simulation results, Fig.2 shows the desired and actual trajectory of attitude angle of base.Fig.3 shows the tracking error of attitude angle of base. Fig.4 and Fig.5 show the desired and actual trajectory of joint angles. Fig.6 shows the tracking errors of joint angles.Fig.7 shows the comparison of the first mode of vibration with active vibration suppression and without active vibration suppression.Fig.8 shows the comparison of the second mode of vibration with active vibration suppression and without active vibration suppression. Fig.9 shows the comparison of end displacement of flexible-arm with active vibration suppression and without active vibration suppression.

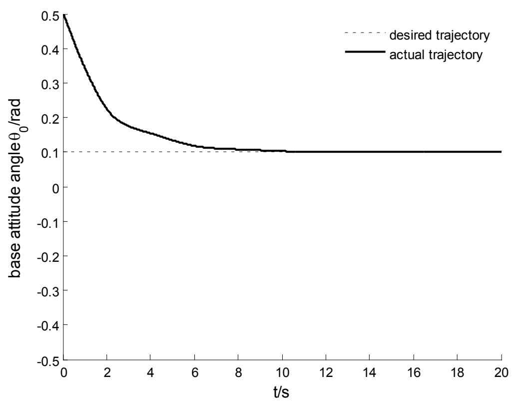

Fig.2 the desired and actual trajectory of attitude angle of base

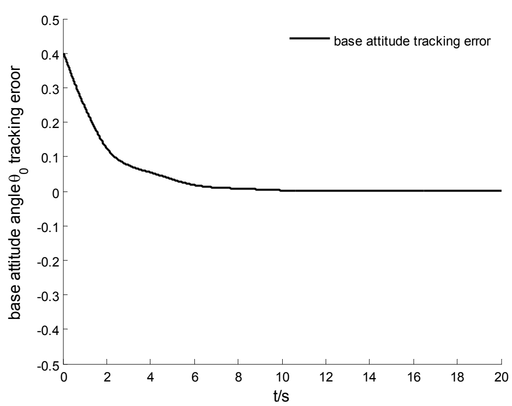

Fig.3 the tracking error of attitude angle of base

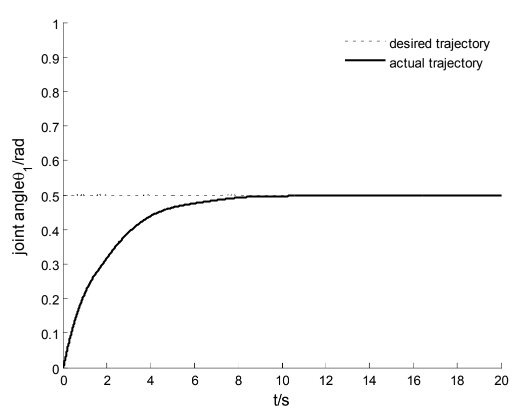

Fig.4 the desired and actual trajectory of joint angle θ_1

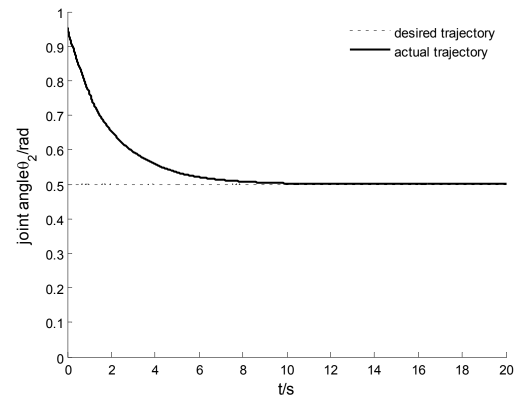

Fig.5 the desired and actual trajectory of joint angle θ_2

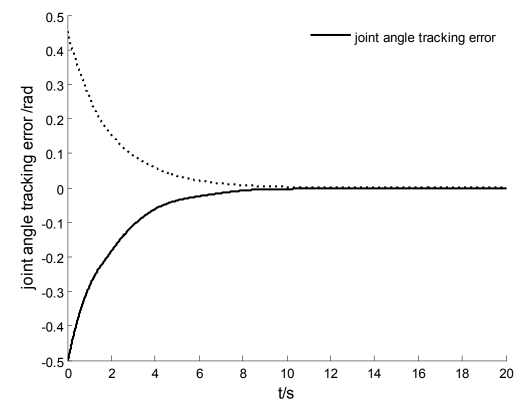

Fig.6 the tracking errors of joint angles

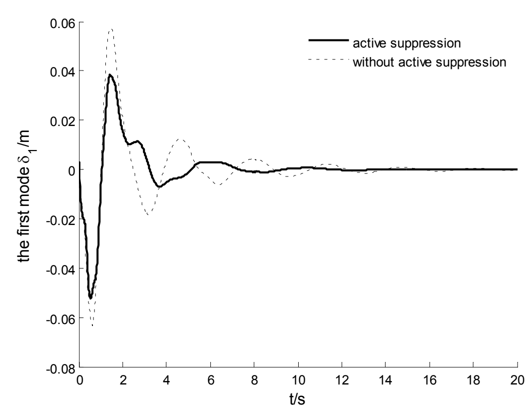

Fig.7 the first mode of vibration (with and without active vibration suppression)

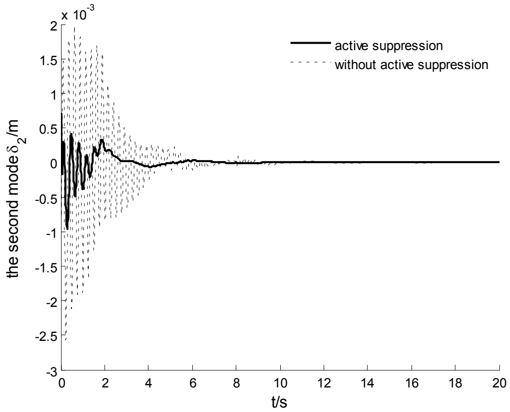

Fig.8 the second mode of vibration (with and without active vibration suppression)

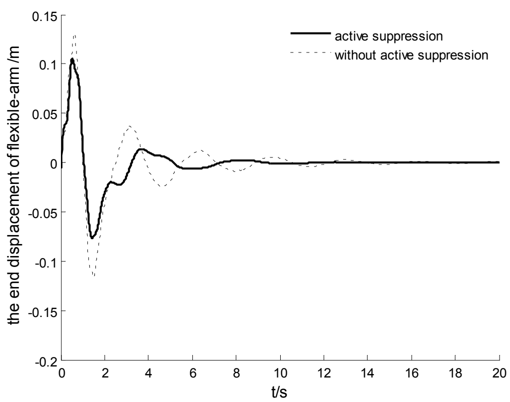

Fig.9 the end displacement of flexible-arm (with and without active vibration suppression)

6 CONCLUSION

In this paper, in order to solve actuator's partial loss and flexible vibration suppression of flexible-arm space robot, a composite control scheme which consists of decentralized terminal sliding mode and PD feedback control is designed to realize fault tolerant control, precious trajectory tracking and flexible vibration suppression based on singular perturbation theory. the results of numerical simulation verify the validity of the proposed control schemes.

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