JOB SCHEDULING FOR ON-ORBIT SPACECRAFT REFUELING THROUGH PLANT GROWTH SIMULATION ALGORITHM

Xiaoyu Zhu [1], Yongqiang Jin [2], Qingzhan Zhang [2], *Bing Qiao [1],

[1] *Nanjing University of Aeronautics and Astronautics, 29 Yudao Street,*

Nanjing 210016, China, E-mail: bqiao@nuaa.edu.cn

[2] *Institute of Aerospace System Engineering Shanghai, 3888 Yuanjiang Road, Minhang District,*

Shanghai 201109, China, E-mail: zh.q.zh@163.com

ABSTRACT

In recent years researchers are seeking the technological means of spacecraft on-orbit refueling, which is one of the most important forms of on-orbit servicing technology, to extend the lifetime of an on-orbit spacecraft. This paper focuses on the job scheduling issues arising from the problem that how to refuel or service multiple spacecraft with small inclinations in a cost-effective manner in which a servicer fetches fuel and sets off to refuel a target spacecraft. The solutions to this specific job scheduling problem are found and optimized through the Plant Growth Simulation Algorithm (PGSA) which is considered as a kind of bionics random searching algorithm which occurs in nature characterizing plant phototropism. In order to verify the effectiveness of the proposed algorithm, computer simulations are conducted and their results are demonstrated.

Keywords: Spacecraft refueling; Lambert problem; plant growth simulation algorithm; optimization

1 INTRODUCTION

The current practice when an on-orbit spacecraft exhaust its onboard fuel is to destroy it by deorbiting it into a graveyard orbit, even though the abandoned spacecraft is still mechanically and electrically functioning well. This will definitely cause the rapid increment of unwanted space debris or other potential troubles for the space activities of human beings. In recent years space scientific professionals are seeking the technological means of spacecraft on-orbit refueling, which is actually one of the most important forms of On-Orbit Servicing (OOS) technology, to extend the lifetime of an on-orbit spacecraft. Servicing and Refueling spacecraft in orbit has the potential to revolutionize spacecraft operations by extending the useful lifetime of the spacecraft, by reducing launching and insurance cost, and by increasing operational flexibility and robustness. Over the past decade, several studies have been conducted, which investigated the relative merit of spacecraft Refueling when compared to replacing them. Moreover, crucial progress in the field of computer science makes it possible to find an optimized sequence and time distribution to refuel all the satellites in the whole constellation.

During the past decades, NASA, the DOD and several other individual organizations or institutes have conducted a number of On-Orbit Servicing studies. A good example of how to implement a servicing vehicle involved with transporting fluids in zero-gravity is introduced in [1] and a conceptual study of refurbishing spacecraft for low cost communications systems is given which shows that refurbishing spacecraft would extend communication services for 20 percent of the cost associated with replacement[2].

Some useful strategies for servicing multiple satellites in GEO is considered in [3], in which the geosynchronous satellites are assumed to have small inclination. The problem is to find the order of visiting the satellites with a single Refueling spacecraft such that the total delta-velocity cost is minimized. We call this the single Refueling spacecraft refueling strategy (RSc). That is, a single service spacecraft plays the role of the sole supplier of fuel[4-6].This kind of refueling strategy have its critical disadvantage that is a failure of the service vehicle in a single-spacecraft scenario will result in the failure of the whole mission.

Recently, an alternative scenario for distributing fuel amongst a large number of satellites has been proposed [7-9]. In this scenario, no single spacecraft is in charge of the complete refueling process. Instead, all satellites share the responsibility of refueling each other on an equal footing. Consequently, it offers a great degree of robustness and protection against failures. For instance, with a P2P strategy a failure of a single spacecraft will have almost no impact on the refueling of the rest of the constellation. A comparison between two distinct baseline refueling scenarios for a satellite constellation in a circular orbit is provided in [10]. It shows that a mixed strategy that incorporates a P2P component may indeed lead to fuel savings as the number of satellites increases. Furthermore, pure peer-to-peer (henceforth abbreviated as P2P) and mixed (combined single-spacecraft and P2P) satellite refueling in circular orbit constellations comprised of multiple satellites are studied which propose an asynchronous P2P strategy that also leads to more efficient refueling [11].

In this paper, a new alternative method for Refueling strategy is proposed. The objective of the scheduling is to find an optimized sequence to refuel all the satellites that have small inclinations in a given mission timespan with minimum delta-velocity cost. The solutions to this specific job scheduling problem are found and optimized through the Plant Growth Simulation Algorithm (PGSA) which is considered as a kind of bionics random searching algorithm which occurs in nature characterizing plant phototropism. In order to verify the effectiveness of the proposed algorithm, computer simulations are conducted and their results are demonstrated.

The rest of the paper, we will present our task into two major parts. In the first part of the current paper we formulate the RSc refueling problem as a minimization problem of a cost function that is a convex combination of the previous two conflicting objectives. In the second and major part of the paper we revisit the refueling problem, with the goal of further improving the transfer costs. We will present a new and efficient approach - PGSA for on-orbit spacecraft refueling that determine the optimal time distribution and sequence with an objective of improving the total delta-velocity cost.

2 PROBLEM DEFINITION

2.1 Refueling problem of constellation

The scenario of on-orbit refueling for a constellation of $S_n(\theta)$ which consists of *n* satellites distributed (perhaps non-uniformly) in a circular orbit with a radius of R_{TSc} can be described as follows: a servicing satellite s_{sc} with the initial mass of $m_{sc}(t_0)$ sets off from an on-orbit fuel station s_0 and radius of R_{SSc} , and a fuel station s_0 with plenty of fuel coplanar circular orbit for a sequence of $R_{SSc} = R_{FS} < R_{TSc}$. In order to simplify the model, we assume that the initial angle of servicing satellite and fuel station to be $\theta_{SSc}(t_0)$ and $\theta_{FS}(t_0)$, where $\theta_{SSc}(t_0) = \theta_{FS}(t_0)$ .The task of the s_{sc} is to service all satellites in the constellation. Therefore, the s_{sc} is required to rendezvous with each of the satellites. After the s_{sc} finishes servicing one satellite, it visits the next satellite until all the satellites have been serviced. Each satellite is visited only once during the servicing mission. As a practical concern, the total time to complete the mission is also specified. The objective is to find the sequence to for s_{sc} to visit all satellites in $S_n(\theta)$ with a given time t_{f} such that the total rendezvous cost ΔV is minimized.

Fig. 1 shows *n*+1 satellites, s_0 and s_1 ,..., s_{n} ($S_n(\theta)$) , in a circular orbit, with the s_{sc} initially at s_0 . Let $\Delta V_{ij}(\mathrm{t}_{ij}, \theta_{ij})$ denote the cost associated with the transfer of s_{sc} from satellite s_i to satellite s_j within a time interval t_{ij} .Note that the cost $\Delta V_{ij}(\mathrm{t}_{ij}, \theta_{ij})$ includes both velocity changes due to the impulses at the initial and terminal points of the transfer orbit from s_i to s_j .The first velocity change will put s_{sc} to a transfer orbit that intersects the orbit of s_j . The second velocity change is necessary in order for s_{sc} to rendezvous with s_j and enter its orbit.

For impulsive transfers the following expression holds between the mass of the satellite just before $(m_{sc}(t_{\mathrm{ij}}^-))$ and just after $(m_{sc}(t_{\mathrm{ij}}^+))$ an impulse at time $t = t_{\mathrm{ij}}$.

$$m_{sc}(t_{\mathrm{ij}}^+) = m_{sc}(t_{\mathrm{ij}}^-)e^{-\Delta V_{ij}/I_{sp}\cdot g_0} \quad (1)$$

where ΔV_{ij} is the gain (or loss) of velocity due to the impulse at $t = t_{\mathrm{ij}}$. For a sequence of $\ell \geq 1$ impulses at times $t_0 \leq t_1 < t_2 < ... < t_\ell$ equation (1) yields.

$$m_{sc}(t_\ell) = m_{sc}(t_0)e^{\sum_{k=1}^{\ell} -\Delta V_k / I_{sp}\cdot g_0} \quad (2)$$

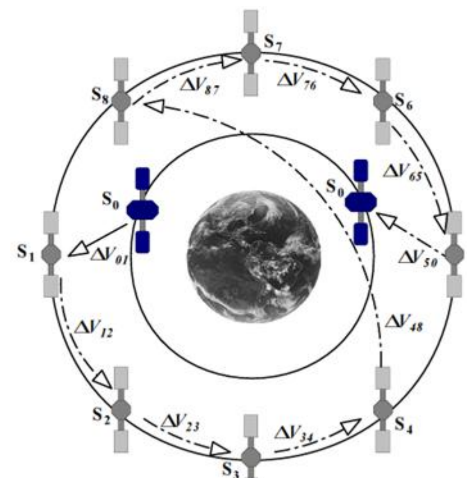


Figure 1. Single Refueling spacecraft refueling strategy

So far we have just reviewed what the optimal single Refueling spacecraft refueling strategy (RSc). Once we obtain the solution to the minimum-cost rendezvous problem between any two satellites on coplanar circular orbits, we are in a position to address the Refueling problem.

2.2 Time-Constrained, Fuel-Optimal, Two-Impulse Transfers

An optimal impulsive rendezvous problem can be defined as the following. Two spacecraft, s_1 and s_2 , are assumed to be orbiting on their respective orbits. The problem is to find a trajectory for s_1 to rendezvous with s_2 such that the total velocity change is minimized. In fact, the optimal impulsive rendezvous problem has been studied for a long time, for instance, several of classical methods are presented in [12] and [13]. By formulating the orbital transfer problem using optimal control theory, the optimal thrusting profile is readily determined from the time history of the primer vector, which is the co-state corresponding to the velocity vector in the adjoint system of equations. The problem even admits a closed-form solution for some special cases. No analytic solutions are known to exist for the general case, however.

In this paper we deal with two-impulse rendezvous for the reason that we can use the solutions to the well-known Lambert problem [14], as shown in Fig.2. Although for some rendezvous problem, multiple-impulse solutions use less fuel than two-impulse solution, they suffer from the drawback of requiring a large amount of computational effort. In addition, the convergence of the algorithms, is not guaranteed, and there are cases where the solutions converge to local minima. Moreover, as it is shown in Ref. 7 the fuel savings gained by the use of three or more impulses may not be as great so as to justify the use of more than two impulses. Most importantly, several efficient methods exist for solving the Lambert problem presented in Ref. 14.

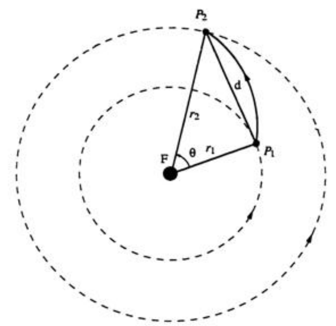


Figure 2. Lambert problem

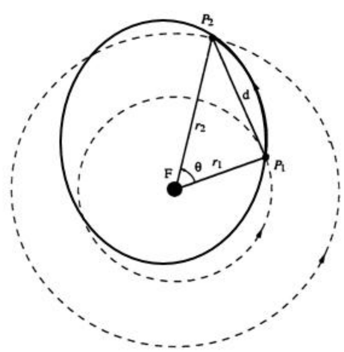


Figure 3. multi-revolution Lambert problem

a. In-Plane Transfer

The total ΔV consists of two components, the plane change and the in-plane transfer for the rendezvous. Concerning the in-plane transfer, using multiple revolution Lambert transfers may result in significant fuel savings which has been shown in [15] recently. The solution to the multi-revolution Lambert problem is more involved than the zero-revolution Lambert problem, however. In fact, the former exhibits a multitude of solutions. As shown in Ref. 15, between any two fixed points, there are actually 2Nmax+1 solutions to the multi-revolution Lambert problem in Fig.3 where Nmax is the number of maximum number of revolutions which are allowed for the chasing satellite. In general, one has to compute all possible 2Nmax+1 candidates before choosing the one that results in the most fuel-efficient transfer (that is, the lowest ΔV).

Reference [16-22] developed an efficient method to quickly calculate the optimal ΔV by comparing only two out of all possible 2*Nmax*+1 candidates. The optimal solution depends on the given transfer time t_f . Figure 3 shows the ΔV vs t_f curve (dashed line) for a transfer between two satellites in the same circular orbit. As shown in Fig. 3, an orbital maneuver between two points strongly depends on the transfer time. Fuel-efficient transfers occur only at distinct points along the ΔV vs t_f curve, namely close to the relative minima of the dashed line in Fig. 4. This sequence of relative minima corresponds to phasing maneuvers (that is, maneuvers that are performed by tangential initial and final burns). The situation can be improved by the use of final coasting arcs. These correspond to horizontal line segments in the ΔV vs. t_f plot.

Therefore, an optimal transfer between satellites in the same circular orbit is composed, in general, by a phasing maneuver plus a final coasting.

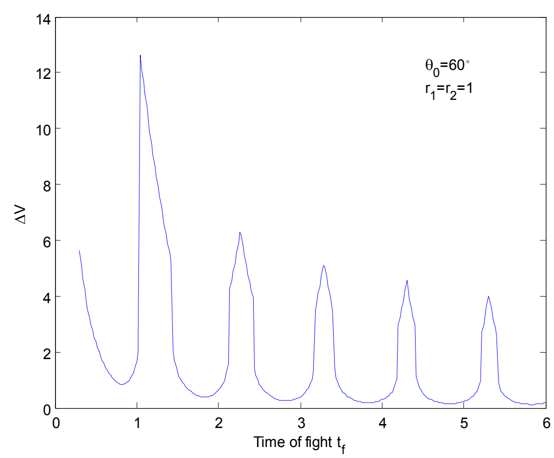


Figure 4. ΔV *vs.* t_f *when* $r_1 = r_2 and \theta_0 = 60^\circ$*, without coasting*

For transfers that include final coasting, the ΔV monotonically decreases as a function of t_f (solid line in Fig. 1). The decrease is not strictly monotonic however, due to the coasting arcs. This creates some complications when trying to compute the optimal time allocation for a multi-segment rendezvous. We will elaborate on this issue later on, when we also propose a solution to the optimal time-allocation problem using integer programming.

In the sequel, a constellation of $n \geq 2$ satellites, $s_i (i = 1,...,n)$ at time t_0 will be denoted by $S_n(\theta(t_0))$, where the vector of initial satellite separation angles defined by $\theta_i(t_0) = v(s_{i+1};t_0) - v(s_i;t_0)(i = 1,...,n-1)$ and $v(s_i;t_0)$ denotes the true anomaly for satellite s_i at initial time t_0 . The satellites are numbered sequentially along the direction of the orbit. Note that, by definition, $\theta_n(t_0) = 2\pi - \sum_{i-1}^{n-1}\theta_i(t_0)$. All angles are measured positive in the direction of the orbit. Since we assume that all satellites are in the same circular orbit, we have that $\theta_i(t) = \theta_i(t_0)$ for all $t = t_0$. We can therefore drop the argument t_0 and write simply $\theta_i(t_0)$. Let also $\theta_{ij} \in [-\pi, \pi]$ denote the lead angle between satellites s_i and s_j . For each ordered pair of satellites $s_i = s_j (i \neq j)$ we assign a time interval t_{ij} within which the rendezvous of satellite s i with satellite s j has to be completed. In the pair (s_i, s_j) satellite s_i will be the active satellite and s_j will be the passive satellite.

Note that if the orbital frequency of the constellation is w_0 then the two-impulse transfer between satellites s_i and s_j can be formulated as a multi-revolution Lambert problem with transfer time t_{ij} and separation angle $\varphi_{ij} = \text{mod}(\theta_{ij} + t_{ij}\omega_0, 2\pi)$.

Subsequently, the moving-target rendezvous problem and the fixed-time ,fixed-endpoint transfer problem are related by the fact that the transfer angle θ defined in the fixed-time, fixed-endpoint rendezvous problem can be written in terms of the initial lead angle θ_0 as $\theta = \theta_0 + t_f\omega_2$ where ω_2 is the orbital frequency of the higher circular orbit of the target spacecraft. Once θ is obtained, the optimal moving-target rendezvous problem becomes a fixed -endpoint transfer problem, and thus it can be solved using the aforementioned procedure.

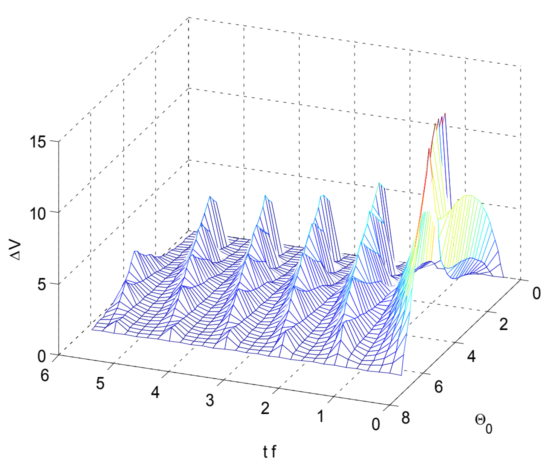


Figure 5. Contour of ΔV *when* $r_1 = r_2 = 1$

With this methodology at hand, for any coplanar circular orbits, minimum ΔV transfer orbits can be obtained for moving-target rendezvous problem with a large range of initial lead angles and transfer time. Thus, a contour plot of the minimum ΔV can be obtained with respect to the lead angles and the transfer time in Fig.5.

Then, let J_{ij} denote the operation associated with the transfer of s_{sc} from satellite s_i to satellite s_j ($i = \{0,1,2,...,n\}$ $j = \{0,1,2,...,n\}; i \neq j$).

Let P_n denote the set of all permutations of the ordered sequence (*1*, *2*,...,*n*). We denote $q \in P_n$ by $q = (q_1, q_2, q_3, ..., q_n)$.

We seek q^* such that the sequence of transfers $s_{q_1^*} \to s_{q_2^*} \to ... \to s_{q_{n-1}^*} \to s_{q_n^*}$ solves the optimization problem

$$\min_{t_{q_iq_{i+1}}} \min_{q \in P_n} \sum_{i=1}^{n-1} \Delta V_{q_iq_{i+1}}(t_{q_iq_{i+1}}, \theta_{q_iq_{i+1}}) \tag{3}$$

Subject to the constraint $\sum_{i=1}^{n-1} t_{q_iq_{i+1}} \leq t_f$.

The optimization parameters are the rendezvous sequence $q \in P_n$ and the corresponding time intervals $t_{q_iq_{i+1}} (i = 1,2,...,n-1)$ for each rendezvous segment. Note that in the formulation of the optimization problem (3)-(4) we have assumed that the service satellite s_{sc} is already at the location of s_0 at the beginning of the refueling process. In other words, the initial cost for s_{sc} to transfer to the first satellite s_0 in the sequence is not taken into account.

This choice simplifies the analysis, and for uniform constellations a circular orbit can be made without loss of generality. A solution to the previous single-vehicle refueling problem for a circular constellation has been proposed in Ref. 7, and involves the solution to the following two problems: (i) the optimal time distribution problem, and (ii) the optimal rendezvous sequence problem. In the optimal time distribution problem, it is assumed that the sequence q is given. Then we solve the problem

$$\min_{t_{q_iq_{i+1}}} \sum_{i=1}^{n-1} \Delta V_{q_iq_{i+1}}(t_{q_iq_{i+1}}) \tag{4}$$

The difficulty in solving the optimal time distribution problem lies in the fact that the functions $\Delta V_{q_iq_{i+1}}(t_{q_iq_{i+1}})$ in (4) are not differentiable with respect to $t_{q_iq_{i+1}}$. In fact, each function $\Delta V_{q_iq_{i+1}}(t_{q_iq_{i+1}})$ consists of several constant segments, due to the final coasting arcs, as seen in Fig. 1. This prevents the use of traditional gradient-based search methods. The solution approach of Ref. 7 then proceeds as follows.

For ease of notation, and without loss of generality, in the remainder of this section we write $\Delta V_{\theta_i}(t_i)$ for the more cumbersome $\Delta V_{q_i q_{i+1}}(t_{q_i q_{i+1}})$.

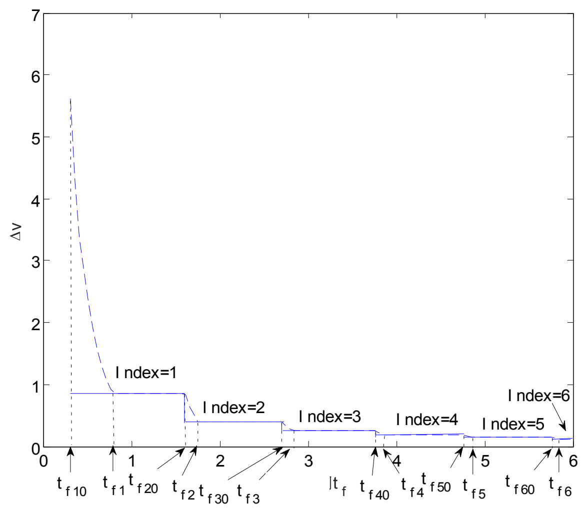


Figure 6. Step function approximation of the cost function.

Inspection of Fig. 6 reveals that each cost function $\Delta V_{\theta_i}(t_i)$ is comprised of a series of constant segments connected by smooth, monotonically decreasing segments. For the i^{th} rendezvous segment, and following the notation of Fig. 6, let $Index_{ij} \quad j = 1,2,..., j_{i,\max}$ denote the costs associated with the constant segments in the function $\Delta V_{\theta_i}(t_i)$ .The upper limit for the index j, $j_{i,\max}$, depends on the maximum time of transfer $t_{i,\max}$ allowed to be distributed to the corresponding segment. A natural choice is to let $t_{i,\max} = t_f$. Similarly, from Fig.6, let $t_{f_{ij}}$, $j = 1,2,..., j_{i,\max}$ denote the times when a curve is followed by a step function, and $t_{f0_{ij}} \quad j = 1,2,..., j_{i,\max}$ denote the time when a step function is followed by a curve.

Typically, $t_{f_{ij}} - t_{f0_{ij-1}}$ is small compared to $t_{f0_{ij}} - t_{f0_{ij-1}}$ for any $j = 1,2,..., j_{i,\max}$, and their difference increases as the transfer time increases.

Based on these observations, we may approximate the cost function for each rendezvous segment by a series of step functions. This is depicted in Fig. 6, where the original cost function is shown in dash lines and the step function approximation is shown in solid lines.

b. In-Plane Transfer

Now, consider two satellites of the set $S_n(\theta)$ with inclination and right ascensions (I_1, Ω_1) and (I_2, Ω_2) [23].

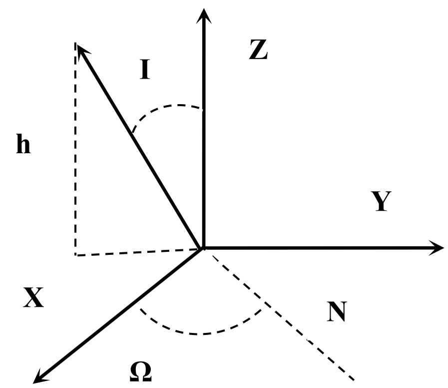


Figure 7. Angular Momentum Projection

Let (X, Y, Z) be an inertial coordinate system with Z along the polar axis, and X and Y define the equatorial plane. As shown in Fig. 7, the coordinates of the projection of the angular momentum vector in the equatorial plane are $h_j \sin I_j (\sin\Omega_j - \cos\Omega_j), j = 1,2$. With equal angular momentum the distance *d* between the two points is

$$\begin{cases} d^2 = h^2 \begin{bmatrix} (\sin I_2 \sin\Omega_2 - \sin I_1 \sin\Omega_1)^2 \\ + (\sin I_2 \cos\Omega_2 - \sin I_1 \cos\Omega_1)^2 \end{bmatrix} \\ d^2 = h^2 \begin{bmatrix} (\sin I_2 - \sin I_1)^2 \\ + 4 \sin I_2 \sin I_1 \sin^2[(\Omega_2 - \Omega_1)/2] \end{bmatrix} \end{cases} \quad (5)$$

For small inclinations this becomes

$$d^2 = h^2\left[(I_2 - I_1)^2 + 4 I_2 I_1 \sin^2(\Delta\Omega/2)\right] \quad (6)$$

Let the angle between the two orbit planes be γ , which is given by

$$\cos\gamma = \cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos\Delta\Omega \quad (7)$$

For small inclinations this becomes

$$\gamma^2 = (I_2 - I_1)^2 + 4 I_2 I_1 \sin^2(\Delta\Omega/2) \quad (8)$$

Thus, the angle between the planes is proportional to the distance between the projections of the angular momentum vectors in the equatorial plane. The ΔV for a plane change of γ is

$$\Delta V = 2v \sin(\gamma/2) \approx v\gamma \quad (9)$$

Thus, the ΔV for the plane change is proportional to the distance between the projections of the angular momentum vectors on the horizontal plane.
Since the satellites are in circular orbits of the same period the in-plane transfer is just an impulse into a small eccentric orbit to change the period to accomplish an orbit phase change, followed by a circularization maneuver. If sufficient

time is allowed this ΔV will be much smaller than the plane change. It may also be possible to combine it with the plane change. In either case the plane change ΔV dominates the total ΔV. Thus, the minimum ΔV is found by minimizing the ΔV for the plane change.

2.3 The Traveling Salesman Problem

Now consider a set of GEO satellites that have small inclinations. The servicing satellite will be referred to as the interceptor and the satellite it is going to rendezvous with will be the target. Since the ΔV for each plane change is proportional to the distance between the projections of the angular momentum vectors on the equatorial plane the minimum ΔV for the plane changes required to visit all the satellites is the minimum path distance through all the projections of the angular momentum vectors on the equatorial plane. This is the Traveling Salesman Problem (TSP), a problem that hasbeen studied extensively. Any point that is far from the other points is one that will require substantial ΔV to reach and can be eliminated from consideration. Since visiting many satellites could require several years one also has to consider the effect of the gravitational perturbations that will result in the projections of the angular momentum vectors slowly changing with time. This makes the problem a Dynamic TSP. In this initial study we will not consider the dynamic TSP.

With this approximation, and for each rendezvous segment, the time interval $[t_{f0_{i0}}, t_{i,\max}]$ is divided into $j_{i,\max}$ subintervals $[t_{f0_{ij-1}}, t_{f0_{ij}}]$ $j = 1,2,...,j_{i,\max}$ The problem of optimal time distribution is now converted into a problem of determining the subinterval $[t_{f0_{ij-1}}, t_{f0_{ij}}]$ in which t_i should be assigned for each i^{th} rendezvous segment, where $1 \le i \le n-1$ .To solve this problem we introduce binary variables x_{ij} , where $i = 1,2,...,n-1$ $j = 1,2,...,j_{i,\max}$ such that

$$x_{ij} = \begin{cases} 1 & if \quad t_i \in [t_{f0_{ij-1}}, t_{f0_{ij}}], j = 1,2,...,j_{i,\max} \\ 0 & \text{otherwise.} \end{cases} \quad ... \quad ...(10)$$

Then the integer program

$$\min_{x_{ij}} \sum_{i=1}^{n-1} \sum_{j=1}^{j_{i,\max}} Index_{ij} x_{ij} \qquad (11)$$

Subject to the constraints

$$\begin{cases} \min \ \sum_{k=1}^{q} \Delta V_{i_k, j_k} & (12.1) \\ s.t. \ nT_{SSc} + pT_{FS} + \sum_{k=1}^{q} T_{i_k, j_k} \le T_{ceil} & (12.2) \\ p = q - n - 1 & (12.3) \end{cases} \qquad (12)$$

The optimal rendezvous sequence problem deals with the determination of the best rendezvous sequence for a given total time.

Given a set of points the TSP is finding the path of minimum length that passes through each of the points. The dynamic TSP (DTSP) is the TSP when each of the points follows a prescribed path. In next section, we will show how to use plant growth simulation algorithm to solve this problem.

3 PLANT GROWTH SIMULATION ALGORITHM (PGSA)

The plant growth simulation algorithm [24-26] is based on the plant growth process, where a plant grows a trunk from its root; some branches will grow from the nodes on the trunk; and then some new branches will grow from the nodes on the branches. Such process is repeated, until a plant is formed. Based on an analogy with the plant growth process, an algorithm can be specified where the system to be optimized first "grows" beginning at the root of a plant and then "grows" branches continually until the optimal solution is found.

By simulating the growth process of plant phototropism, a probability model is established. In the model, a function g(Y) is introduced for describing the environment of the node Y on a plant. The smaller the value of g(Y), the better the environment of the node for growing a new branch. The outline of the model is as follows: A plant grows a trunk M, from its root B_0 . Assuming there are k nodes $B_{M1}, B_{M2}, B_{M3}, ..., B_{Mk}$ that have better environment than the root on the trunk M, which means the function g(Y) of the nodes and satisfy $g(B_{Mi}) < g(B_0)$ then morphactin concentrations $C_{M1}, C_{M2}, C_{M3}, ..., C_{Mk}$ of nodes $B_{M1}, B_{M2}, B_{M3}, ..., B_{Mk}$ are calculated using

$$\begin{cases} C_{Mi} = \dfrac{g(B_0) - g(B_{Mi})}{\Delta_1} \\ \Delta_1 = \sum_{i=1}^{k} (g(B_0) - g(B_{Mi})) \end{cases} \qquad (13)$$

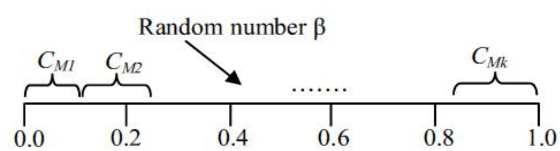


Figure 8. Morphactin concentration state space

The significance of equation (13) is that the morphactin concentration of a node is not only dependent on its environmental information but also depends on the environmental information of the other nodes in the plant, which really describes the relationship between the morphactin concentration and the environment. From (13), we can derivate $\sum_{i=1}^{k} C_{Mi} = 1$, which means that the morphactin concentrations $C_{M1}, C_{M2}, C_{M3}, ..., C_{Mk}$ of nodes $B_{M1}, B_{M2}, B_{M3}, ..., B_{Mk}$ form a state space shown in Fig. 7. Selecting a random number β in the interval [0, 1], β is like ball thrown to the interval [0, 1] and will drop into one of $C_{M1}, C_{M2}, C_{M3}, ..., C_{Mk}$ in Fig. 7, then the corresponding node that is called the preferential growth node will take priority of growing a new branch in the next step. In other words, B_{MT} will take priority of growing a new branch if the selected β satisfies $0 \le \beta \le \sum_{i=1}^{T} C_{Mi} (T = 1)$ or $\sum_{i=1}^{T-1} C_{Mi} < \beta < \sum_{i=1}^{T} C_{Mi} (T = 2,3,...,k)$. For example, if random number β drops into C_{M2} , which means $\sum_{i=1}^{1} C_{Mi} < \beta < \sum_{i=1}^{2} C_{Mi}$, then the node B_{M2} will grow a new branch *m*. Assuming there are *q* nodes $B_{m1}, B_{m2}, B_{m3}, ..., B_{mq}$ which have a better environment than the root B_0 on the branch *m*, and their corresponding morphactin concentrations are $C_{m1}, C_{m2}, C_{m3}, ..., C_{mq}$ Now, not only the morphactin concentrations of the nodes on branch m need to be calculated, but also the morphactin concentrations of the nodes except B_{M2} (the morphactin concentration of the node B_{M2} becomes zero after growing the branch *m*) on trunk *M* need to be recalculated after growing the branch *m*. The calculation can be done using (9), which is gained from (8) by adding the related terms of the nodes on branch m and abandoning the related terms of the node B_{M2}

$$\begin{cases} C_{Mi} = \dfrac{g(B_0) - g(B_{Mi})}{\Delta_1 + \Delta_2} (i = 1,3,...,k) \\ C_{mi} = \dfrac{g(B_0) - g(B_{mi})}{\Delta_1 + \Delta_2} (j = 1,2,...,q) \end{cases} \quad (14)$$

Where

$$\begin{cases} \Delta_1 = \sum_{i=1, i\neq 2}^{k} (g(B_0) - g(B_{Mi})) \\ \Delta_2 = \sum_{j=1}^{q} (g(B_0) - g(B_{mj})). \end{cases} \quad (15)$$

We can also derivate $\sum_{i=1, i\neq 2}^{k} C_{Mi} + \sum_{j=1}^{q} C_{mj} = 1$ from (9). Now, the morphactin concentrations of the nodes (except B_{M2}) on trunk *M* and branch m will form a new state space (The shape is the same as Fig. 7, only the nodes are more than that in Fig. 7). A new preferential growth node, on which a new branch will grow in the next step, can be gained in a similar way as B_{M2} Such process is repeated until there is no new branch to grow, and then a plant is formed.

4 OPTIMAL SERVICING THROUGH PLANT GROWTH SIMULATION ALGORITHM

The candidate nodes for optimal rendezvous sequence are determined using the minimum ΔV (or the loss of fuel). The estimation of these candidate nodes basically helps in reduction of the search space for the optimization procedure.

$$\min_{x_{ij}} \sum_{i=1}^{n-1} \sum_{j=1}^{j_{i,\max}} Index_{ij} x_{ij} \quad (11)$$

From the viewpoint of optimal mathematics, the nodes on a plant can express the possible solutions; *g*(Y) can express the objective function; the length of the trunk and the branch can express the search domain of possible solutions; the root of a plant can express the initial solution; the preferential growth node corresponds to the basic point of the next searching process. In this way, the growth process of plant phototropism can be applied to solve the problem of integer programming.

A complete algorithm for the proposed method of On-Orbit Servicing is given below:

1. Input the system data such as coding mode and traversal of the order of the satellite of the multi-criteria travelling salesman system, constraints limits etc. Ensure that each satellite is passed through only once and any one of the subaggregate will not form a complete TSP circuit;
2. Form the search domain by giving the range of On-Orbit Servicing available which corresponds to the length of the trunk and the branch of a plant;
3. Give the initial solution X_0 (X_0 is vector) which corresponds to the root of a plant, and calculate the initial value objective function (fuel loss or ΔV);
4. Let the initial value of the basic point X^b , which corresponds to the initial preferential growth node of a plant, and the initial value of optimization X^{best} equal to X_o .
5. Let F^{best} that is used to save the objective function value of the best solution X^{best} be equal to $f(X_o)$,namely, $X^b = X^{best} = X_o$ and $F^{best} = f(X_o)$;
6. Initialize iteration count, *i=1*;
7. For *j=n* to *m*(with step size 1), where *m* is the minimum available size and *n* is maximum available size;
8. Search for new feasible solutions;
9. For each solution X^b in step 8;

10. If the ΔV constraints is satisfied go to step 10; otherwise abandon the possible solution X^b and go to step 12;

11. Calculate $f(X^b)$ for each solution of X^b in step 8 and compare with $f(X_o)$ .Save the feasible solutions if $f(X^b)$ less than $f(X_o)$; Otherwise go to step 12;

12. If $i \geq N_{\max}$ go to step 16; otherwise go to step 14;

13. Calculate the probabilities $C_1, C_2, C_3, ..., C_k$ of feasible solutions $X_1, X_2, X_3, ..., X_k$, by using equation (14), which corresponds to determining the morphactin concentration of the nodes of a plant;

14. Calculate the accumulating probabilities $\sum C_1, \sum C_2, ..., \sum C_k$ of the solutions $X_1, X_2, X_3, ..., X_k$. Select a random number β from the interval [0 1], β must belong to one of the intervals $[0, \sum C_1], (\sum C_1, \sum C_2], ..., (\sum C_{k-1}, \sum C_k]$, the accumulating probability of which is equal to the upper limit of the corresponding interval, and it will be the new basic point X^b for the next iteration, which corresponds to the new preferential growth node of a plant for next step;

15. Increment *i* by *i+1* and return to step 6;

16. Output the results and stop.

5 SIMULATION & RESULTS

In this example, we consider a constellation with n = 20 evenly distributed satellites in a GEO orbit provided by NASA. Two possible refueling scenarios are compared. These scenarios are depicted in Table. 1.

Table 1: Orbital elements provided by NASA

Object	Name	Inclination(deg)	Rt. Ascension(deg)	Longitude(deg)
16274	Morelos 2	0.9128	90.595	197.63
19772	Intelsat VA F15	1.7156	87.1401	221.59
19883	TDRS 4	2.3935	82.265	68.81
15677	Gstar 1	2.6347	80.6943	169.65
22316	IUS-13 SRM-2	4.1882	43.4184	130.83
21641	IUS-15	6.3096	70.5983	85.52
	SRM-2			
13595	Intelsat V F5	7.5253	49.9744	277.91
12089	Intelsat V F2	8.2849	46.1791	96.56
12474	Intelsat 501	8.7881	44.4057	28.91
12472	GOES 5	9.9086	39.2166	354.07
13636	DSCS II F-16	10.0680	41.2597	146.77
13643	IUS-2 SRM-2	11.2379	37.5653	287.66
15236	Leasat 2	11.8924	40.1735	168.11
12046	FLTSATCOM F4	12.2709	31.7337	28.04
11621	DSCS II F-13	12.6462	33.5862	127.88
11144	FLTSATCOM F5	13.0725	55.3282	81.67
10669	DSCS II F-11	13.0939	32.6799	191.45
10000	FLTSATCOM F1	14.0176	26.7312	274.49
11256	SCATHA	14.4197	43.9506	309.11
12635	DSCS II F-7	15.2555	25.2461	203.62

Figure 9 and Figure 10 shows the iterative process of PGSA and ACS (Ant colony algorithm) for this problem. F(X) represents the path of minimum length in TSP. We can see that though ACS uses less number of iteration, PGSA reduces time consumption a lot.

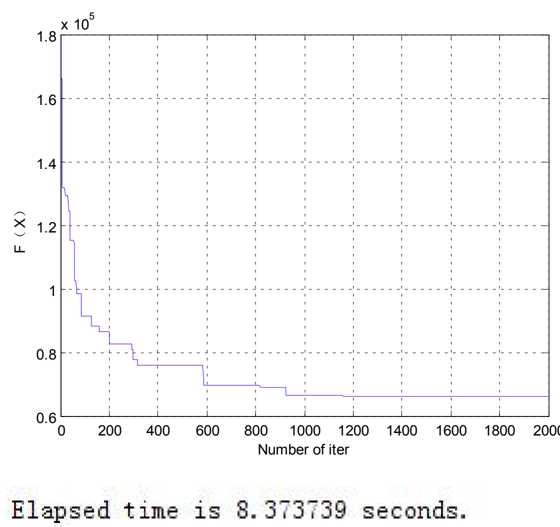
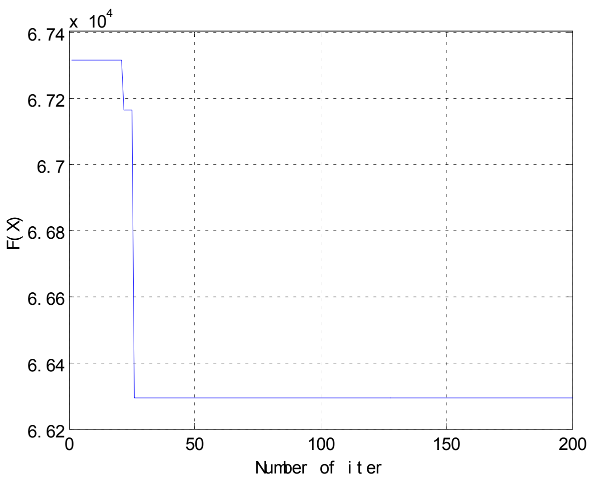


Figure 9. Time and Number of Iteration for PGSA simulation

Figure 11 shows that except for several points we can visually determine most of the minimum distance path through all the points. Although not presented here we selected several sets of objects from the Space Object Catalog and we found that in each case we could visually determine much of the minimum distance path. Since much of the solution can be visually determined the remainder of the solution was obtained by trial and error. The solution is given in Figure 12. Figure 12 shows the order of space object longitude distribution. Figures 13 and show the total ΔV along with the amount for the plane change (blue bar) and in-plane transfer (red bar) that required for rendezvous in 30 days.

Comparing with the results for servicing multiple satellites in GEO is considered in [3], both of them are in good performance in dealing with the problem. In addition, the optimization process of PGSA is smoother, faster. Ant colony algorithm is less efficient, but it can give a relatively large number of non-inferior solution solutions.



Elapsed time is 14.304091 seconds.

Figure 10. Number of Iteration for ACS simulation

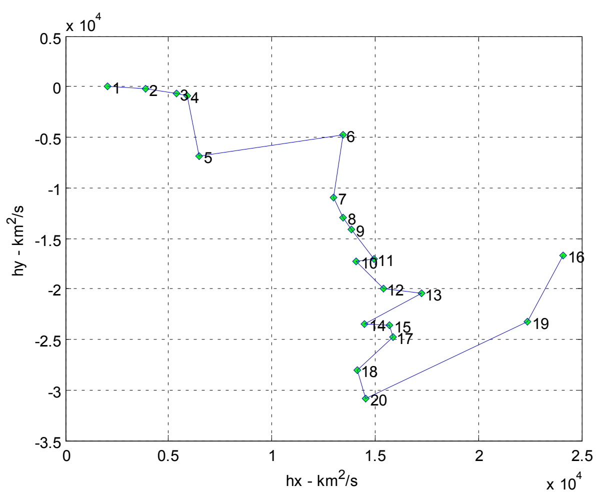


Figure 11. Minimum ΔV Solution

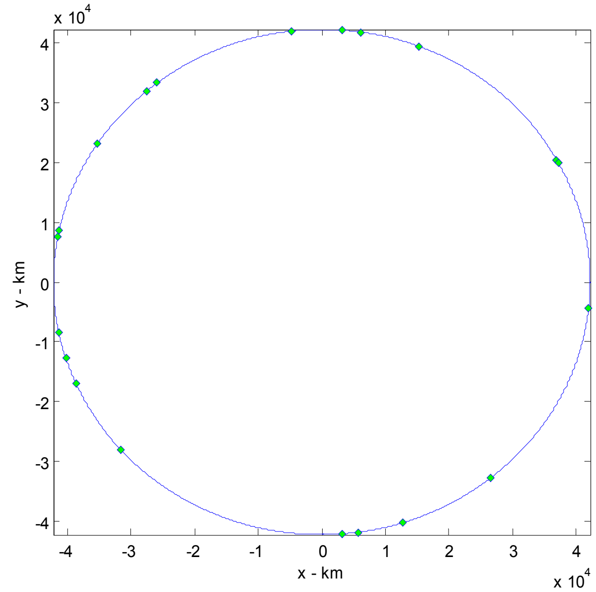


Figure 12. Space Object Longitude Distribution

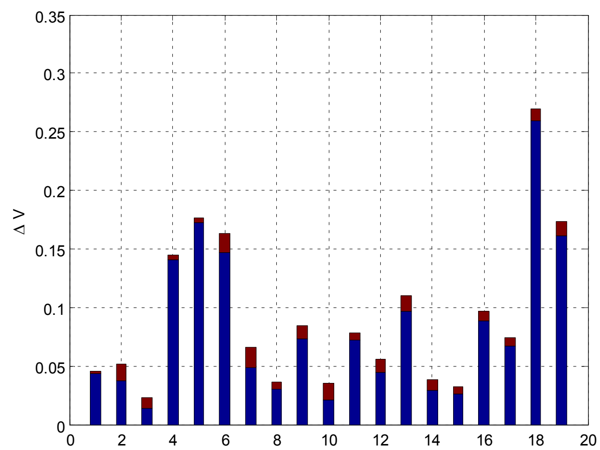


Figure 13. Total ΔV

6 CONCLUSION

This paper addresses the problem of how to optimally rendezvous with a set of satellites in geosynchronous orbit that have small inclinations and give a new alternative scenario based on plant growth simulation algorithm (PGSA) for Refueling strategy. Numerical examples indicate that the latter, mixed strategy, may lead to fuel savings when the number of satellites is large. Needless to say, delivery/redistribution of fuel is only one case where a mixed rendezvous strategy can be beneficial. Other cases include resupply of consumables, service and repair missions, avionics upgrades, etc. In all these cases, the results of this paper indicate that a distributed delivery of consumables can lead to reduced delivery costs. The global optimization and the non-limitation of the parameter setting performance of PGSA are all favorable to extend the theory to the practical problems in the field of combinatorial optimization.

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