Theoretical Analysis of Triangle Matching Method Based on Craters for Spacecraft Localization

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ABSTRACT

This paper aims to derive advantage of triangle matching method for spacecraft localization by theoretical analysis based on Triangle Similarity Matching (TSM) method, and validates that by the experiment. Concretely, TSM estimates the own location by matching craters in a map of the moon and an image taken by the aircraft. This paper focuses on inner and cross products in the mechanism of TSM, and analyzes that. From the theoretical analysis, we can derive two things: (1) if the only craters outside of the triangles are in the different locations, the inner and the cross products' values do not relate on the shape; (2) since the inner and the cross products are influenced of the vector's length, the products should be divided by the length; and (3) the triangles have not to be congruence, but they have to be similar for the advantage of the triangle shape. Furthermore, we improve TSM based on the above findings, and apply it to aircraft localization problems as experiments to validate the effectiveness. From the experimental results, we revealed those things: (a) the modified TSM can perform accurately than any other methods; (b) the modified TSM can decrease the distance from the true location, averagely 0.3, and max 9.52; and (c) the modified TSM can estimate the location in the difficult situations by matching similar triangles.

1 INTRODUCTION

Lately, planetary exploration missions have become concrete because of technological development. For example, Kaguya satellite was launched by JAXA in order to find a clue for the origin of the moon and verify the techniques for orbiting and controlling it around the moon. It could be successfully launched into orbit and collected the data of surface on the moon for about two years [1]. Lately, the planetary exploration missions become advanced and complex based on these data. Especially, space probes which explore the surface of the planet are required (*e.g.*, Curiosity). Since the space probe cannot explore and move to any area of the planet because of limited energy source, the aircraft mounting that has to land on near the valuable area accurately, but this is not achieved yet. From this reason, it is a significant issue for a spacecraft to land a near target towards effective and efficient planetary exploration because it is hard for a spacecraft to explore an area which is far from a landing location. For this purpose, Japan Aerospace Exploration Agency (JAXA) has planned the Smart Lander for Investigating Moon (SLIM) mission which aims at establishing the pin-point spacecraft landing technology [2]. In this mission, the spacecraft mounts the crater database which includes the locations of craters on the moon obtained from Kaguya satellite, and estimates the spacecraft current location through the following three procedures: (1) the spacecraft takes the camera shot image on the moon; (2) it extracts the craters from the camera shot image, and (3) it estimates its own location by comparing the extracted craters with those in the crater database. For the above matching issue, JAXA proposed the line segment matching method (named LSM in this paper) [3], while we proposed the Triangle Similarity Matching (TSM) method [4]. Although both matching methods utilize the shapes formed by connecting craters as dots, which shapes is effective for the matching is unclear. To clarify it, we aim at analyzing the TSM method theoretically from the viewpoint of the spacecraft localization. In addition, TSM is effective, but this is empirical. This suggests that TSM is not guaranteed for the effectiveness, and the theoretical analysis are required for TSM. For this purpose, we focus on the situation where the craters' location has some differences even if the craters in the camera shot image correspond to those in the crater database. Note that such differences are often occurred by many reasons including an altitude of a spacecraft, the brightness of camera shot image, and so on.

This paper organized as follows. Section 2 explains TSM as a background knowledge, and Section 3 explains the detail of the theoretical analysis in this paper. Next, the modified TSM based on the theoretical analysis is introduced in Section 4. Experiment to investigate the effectiveness of the modified TSM and validate the analysis in Section 5. Finally, our conclusion is given in Section 6.

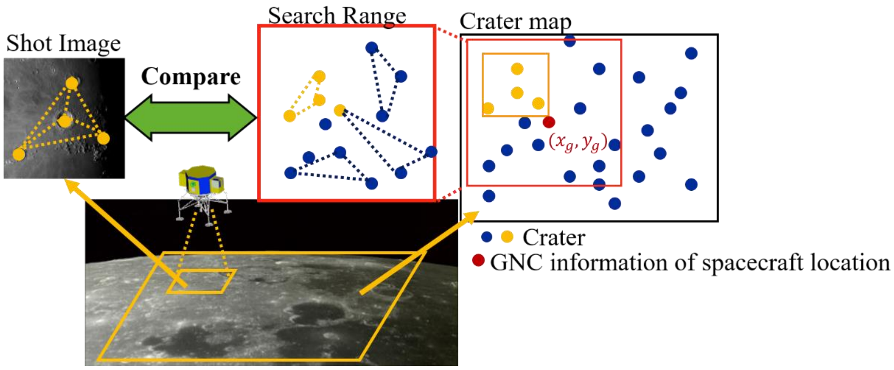

Figure 1. Triangle Similarity Matching

2 BACKGROUND

2.1 General Framework

A spacecraft has a camera shot image and a crater map for the spacecraft localization. Generally, a spacecraft estimates the own location by matching craters between the camera shot image and the crater map. The camera shot image is taken by the spacecraft before the estimation. The crater map is already made and mounted into the spacecraft. In addition, Triangle Similarity Matching (explained below) has a triangle database. There are triangles' data in the triangle database, mainly coordinates of three craters forming a triangle, cosines of the triangle, and length of all sides of the triangle.

2.2 Triangle Similarity Matching (TSM)

Triangle Similarity Matching (TSM) method is a crater matching method based on triangles [3]. Figure 1 shows an overview of TSM, and Figure 2 shows a method flow. First, TSM creates triangles from the craters of the camera shot image. Next, TSM selects two triangles from the crater map and the camera shot image (one is in the crater map, the other is in the camera shot image.) After that, TSM compares the triangles for similar or not, and searches the craters among the craters not consisting the triangles. In these processes, if the triangles are not matched or the number of the matched craters is under a threshold, TSM returns to the process for selecting the triangles from the crater map and the camera shot image. On the other hand, if the triangles are matched and the number of the matched craters is over the threshold, TSM estimates the own location by calculating the coordinates of the matched triangle and the matched craters. Since this paper focuses on the processes of searching the triangles and the craters, we explain the detail of these processes below.

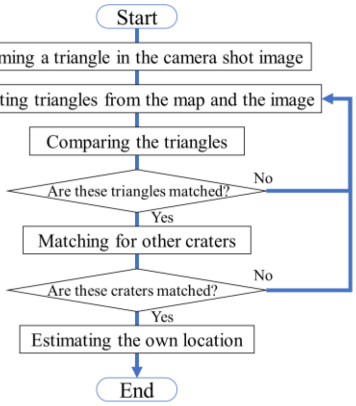

Figure 2. Method flow

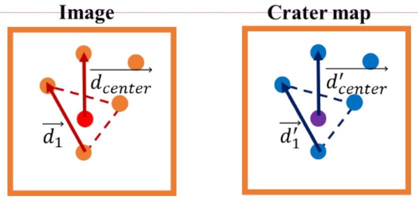

Figure 3. pairing

The searching triangle process utilizes cosines of the angles and the length of the triangles. First, TSM compares all sides between the two triangles: the ratio of the sides has to be from 0.8 to 1.2. In addition, TSM calculates the difference between the angles of the triangles by Equation (1). In this equation, $\boldsymbol{i}$ indicates each vertex of the triangles, $\boldsymbol{\theta_i}, \boldsymbol{\theta'_i}$ are the angles of the vertex $\boldsymbol{i}$, and $\boldsymbol{DIFF}$ is a threshold. TSM determines the triangles are matched by the two terms.

Comment [上野1]: [4]

$$\sum_{i=1}^{3} |\boldsymbol{cos\theta_i} - \boldsymbol{cos\theta'_i}| < \boldsymbol{DIFF} \qquad (1)$$

The searching craters process (called "pairing" in this paper) utilizes an inner and a cross products to determine whether the craters are matched or not. Figure 3 shows an example of the pairing. The left side and the right side indicate the situations of the camera shot image and the crater map, respectively. There are one triangle, five craters, and two arrows in each side of this figure. Note that the circle in the center of the triangle indicates a center of balance of this triangle. The arrows show vectors: one is the vector along to the long side of the triangle, the other is that connected from the center of balance to the crater in outside of the triangle. TSM selects all craters in outside of the triangles from the map and the image (*i.e.*, two kinds of craters are selected in each side of this figure), and determines the craters are matched or not based on the inner and the cross products of the two vectors by the pairing. Concretely, TSM determines that by following terms. Inequalities (2) and (3) show the terms of the inner and the cross products, respectively. Equation (3) shows $\boldsymbol{\gamma}$ in both inequalities. *MIND2* is a threshold. This indicates the allowable amount of the difference of the craters' location.

$$\boldsymbol{I} = \left| \overrightarrow{\boldsymbol{d_1}} \cdot \overrightarrow{\boldsymbol{d_{center}}} - \frac{\overrightarrow{\boldsymbol{d'_1}} \cdot \overrightarrow{\boldsymbol{d'_{center}}}}{\boldsymbol{\gamma}^2} \right| < \boldsymbol{MIND2} \qquad (2)$$

$$\boldsymbol{C} = \left| \overrightarrow{\boldsymbol{d_1}} \times \overrightarrow{\boldsymbol{d_{center}}} - \frac{\overrightarrow{\boldsymbol{d'_1}} \times \overrightarrow{\boldsymbol{d'_{center}}}}{\boldsymbol{\gamma}^2} \right| < \boldsymbol{MIND2} \qquad (3)$$

$$\boldsymbol{\gamma} = \left| \frac{\overrightarrow{\boldsymbol{d_1}}}{\overrightarrow{\boldsymbol{d'_1}}} \right| \qquad (4)$$

3 THEORETICAL ANALYSIS

3.1 Premise

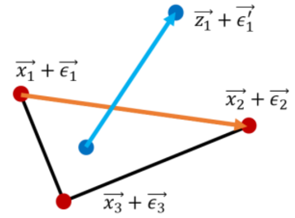

Figure 4. Premise

This paper considers triangle shape's advantage by analyzing the pairing mechanism in TSM. Figure 4 shows a premise for the considering. In this figure, there are four craters, one triangle, and two vectors. $\overrightarrow{\boldsymbol{x_1}}$, $\overrightarrow{\boldsymbol{x_2}}$, $\overrightarrow{\boldsymbol{x_3}}$, $\overrightarrow{\boldsymbol{z_1}}$ indicate position vectors, and $\overrightarrow{\boldsymbol{\epsilon_1}}$, $\overrightarrow{\boldsymbol{\epsilon_2}}$, $\overrightarrow{\boldsymbol{\epsilon_3}}$, $\overrightarrow{\boldsymbol{\epsilon'_1}}$ indicate gaps between craters in the crater map and the camera shot image. In addition, we assume that $\boldsymbol{\gamma} \approx \boldsymbol{1}$ because TSM selects similar triangles before the pairing.

3.2 Difference of the outside craters

At first, we consider only the difference of the craters outside of the triangle. From the inequalities (2) and (3), the inner and the cross products are calculated as Eq. (5) in the case of no difference, and the differences of both products are as calculated as Eq. (6) if only $\overrightarrow{\epsilon'_1}$ exists. In particular, Eq. (6) suggests that the error of the point outside of the triangle is not affected by the shape if the long sides have no error. Furthermore, the differences of both products are calculated in Eq. (7) if only $\overrightarrow{\epsilon_1}$ and $\overrightarrow{\epsilon_2}$ exist, where $\overrightarrow{\epsilon_{12}} = \overrightarrow{\epsilon_2} - \overrightarrow{\epsilon_1}$. In this situation, if the term including $\overrightarrow{x_3}$ becomes a negative value, the triangle shape has an advantage over the line shape. Since cross products cannot become negative values, the triangle shape has an advantage of $-\frac{1}{3}\overrightarrow{\epsilon_{12}} \times \overrightarrow{x_3}$ in the cross products of Eq. (6). On the other hand, since inner products might become negative values, if $\overrightarrow{\epsilon_{12}}\overrightarrow{x_3} > 0$, the triangle shape has the advantage in the inner product. From these results, the shape is important for the matching when the outside craters are matched.

$$\begin{cases} (\overrightarrow{x_2} - \overrightarrow{x_1})\left(\overrightarrow{z_1} - \frac{\overrightarrow{x_1}+\overrightarrow{x_2}+\overrightarrow{x_3}}{3}\right) \\ (\overrightarrow{x_2} - \overrightarrow{x_1}) \times \left(\overrightarrow{z_1} - \frac{\overrightarrow{x_1}+\overrightarrow{x_2}+\overrightarrow{x_3}}{3}\right) \end{cases} \qquad (5)$$

$$\begin{cases} \left|(\overrightarrow{x_2} - \overrightarrow{x_1})\overrightarrow{\epsilon'_1}\right| \\ \left|(\overrightarrow{x_2} - \overrightarrow{x_1}) \times \overrightarrow{\epsilon'_1}\right| \end{cases} \qquad (6)$$

$$\begin{cases} \left|-\frac{1}{3}(|\overrightarrow{\epsilon_2}|^2 - |\overrightarrow{\epsilon_1}|^2 + \overrightarrow{\epsilon_{12}}\overrightarrow{x_3}) + \overrightarrow{\epsilon_{12}}\overrightarrow{z_1}\right| \\ \left|-\frac{1}{3}\overrightarrow{\epsilon_{12}} \times \overrightarrow{x_3} + \overrightarrow{\epsilon_{12}} \times \overrightarrow{z_1}\right| \end{cases} \qquad (7)$$

3.2 Difference of all craters

If all differences exist, the inner and the cross products are calculated by Equations (8) and (9).

$$\left|\overrightarrow{\epsilon_{12}}\left(\overrightarrow{z_1} - \frac{1}{3}(2\overrightarrow{x_2} + \overrightarrow{x_3})\right) - \frac{1}{3}\overrightarrow{\epsilon_3}(\overrightarrow{x_2} - \overrightarrow{x_1} + \overrightarrow{\epsilon_{12}}) + \frac{2}{3}\overrightarrow{\epsilon_1}(\overrightarrow{x_2} - \overrightarrow{x_1}) - \frac{|\overrightarrow{\epsilon_2}|^2 + |\overrightarrow{\epsilon_1}|^2}{3}\right| \qquad (8)$$

$$\left|\overrightarrow{\epsilon_{12}} \times \left(\overrightarrow{z_1} - \frac{1}{3}\overrightarrow{x_3}\right) - \frac{1}{3}\overrightarrow{\epsilon_3} \times (\overrightarrow{x_2} - \overrightarrow{x_1} + \overrightarrow{\epsilon_{12}}) - \frac{2}{3}(\overrightarrow{\epsilon_2} \times \overrightarrow{x_2} - \overrightarrow{\epsilon_1} \times \overrightarrow{x_1})\right| \qquad (9)$$

Equations (10) and (11) are extracted from Eqs. (8) and (9) for the terms of $\overrightarrow{x_3}$ and $\overrightarrow{\epsilon_3}$. These equations indicate the differences generated by the triangle shape. If the triangle shape has the advantage, Eqs. (10) and (11) become 0 or negative values. We consider only Eq. (10) below.

$$\left|-\frac{1}{3}\overrightarrow{\epsilon_{12}} \cdot \overrightarrow{x_3} - \frac{1}{3}\overrightarrow{\epsilon_3}(\overrightarrow{x_2} - \overrightarrow{x_1})\right| \qquad (10)$$

$$\left|-\frac{1}{3}\overrightarrow{\epsilon_{12}} \times \overrightarrow{x_3} - \frac{1}{3}\overrightarrow{\epsilon_3} \times (\overrightarrow{x_2} - \overrightarrow{x_1})\right| \qquad (11)$$

If Eq. (10) is 0, the relationship of the differences of the outside crater and the long side of the triangle is derived through Equation (12)-(15). In these equations, θ is an angle between vectors $\overrightarrow{x_3}$ and $\overrightarrow{\epsilon_{12}}$, and θ' is an angle between $\overrightarrow{x_2} - \overrightarrow{x_1}$ and $\overrightarrow{\epsilon_3}$.

$$-\frac{1}{3}\overrightarrow{\epsilon_{12}}\overrightarrow{x_3} - \frac{1}{3}\overrightarrow{\epsilon_3}(\overrightarrow{x_2} - \overrightarrow{x_1}) = 0 \qquad (12)$$

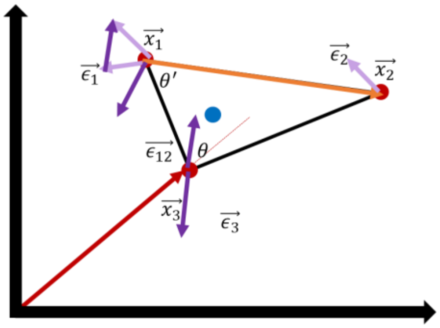

Figure 5. Term of Equation (15)

$$-\frac{1}{3}\overrightarrow{\epsilon_{12}x_3} = \frac{1}{3}\overrightarrow{\epsilon_3}(\overrightarrow{x_2} - \overrightarrow{x_1}) \qquad (13)$$

$$|\overrightarrow{\epsilon_{12}}| = -\frac{\overrightarrow{\epsilon_3}(\overrightarrow{x_2}-\overrightarrow{x_1})}{|\overrightarrow{x_3}|cos\theta} \qquad (14)$$

$$|\overrightarrow{\epsilon_{12}}|cos\theta = -\frac{|\overrightarrow{x_2}-\overrightarrow{x_1}|}{|\overrightarrow{x_3}|}|\overrightarrow{\epsilon_3}|cos\theta' \qquad (15)$$

Figure 5 shows the term of Equation (15). In Equation (15), $\frac{|\overrightarrow{x_2}-\overrightarrow{x_1}|}{|\overrightarrow{x_3}|}$ can adjust the difference between the scales of $\overrightarrow{\epsilon_3}$ and $\overrightarrow{\epsilon_{12}}$. In addition, each cosine maps the differences into each vector as an axis. Note that we can calculate in the same manner of that when we consider Eq. (11). From this equation, $\overrightarrow{\epsilon_3}$ and $\overrightarrow{\epsilon_{12}}$ are point-symmetry for the advantage of the triangle shape. This suggests that the process of the selecting the similar triangle in TSM is important for that.

4 IMPLEMENTATION

We modify TSM based on the theoretical analysis, the searching triangle and the paring processes. From Eqs. (2) and (3), the values of the inner and cross products are influenced by the length of the long side of the triangle, *i.e.*, if the long side is large, the difference of the craters has to be small for the matching; otherwise, that has not to be small. To solve this issue, we improve TSM by changing Eqs. (2) and (3) to Eqs. (16) and (17). Eqs. (16) and (17) is that the threshold becomes $|\overrightarrow{\boldsymbol{d_{center}}}| * \boldsymbol{MIND2}$ in Eqs. (2) and (3).

$$\boldsymbol{I} < \boldsymbol{MIND2} * |\overrightarrow{\boldsymbol{d_{center}}}| \qquad (16)$$

$$\boldsymbol{C} < \boldsymbol{MIND2} * |\overrightarrow{\boldsymbol{d_{center}}}| \qquad (17)$$

Note that the latest TSM (utilized in Experiment of this paper) is modified for the paring. Concretely, it utilizes $\boldsymbol{I^2} + \boldsymbol{C^2}$ instead of Eqs. (2) and (3). Equation (17) shows the term to match the craters. $\boldsymbol{MIND2'}$ is a threshold being different from $\boldsymbol{MIND2}$. From this reason, we utilize Eq. (18) for the latest TSM.

$$\boldsymbol{I^2} + \boldsymbol{C^2} < \boldsymbol{MIND2'} \qquad (17)$$

$$\boldsymbol{I^2} + \boldsymbol{C^2} < \boldsymbol{MIND2'} * |\overrightarrow{\boldsymbol{d_{center}}}| \qquad (18)$$

On the other hand, Eq. (15) shows that the triangles do not have to be congruence, and they should be similar for the matching. This suggests that TSM's terms for the searching triangle are not required. We improve TSM based on this: we remove the terms for the length of all sides of the triangles, and change Eq. (1) to Eq. (19) in this process. In Equation (19), $\boldsymbol{DIFF'}$ indicates a threshold being different from $\boldsymbol{DIFF}$. In addition, the summation of two angles is compared, unlike Eq. (1). The two angles are not the maximum angle in the triangle. From this improvement, TSM searches the similar triangles.

$$\sum_{i=1}^{2}|\boldsymbol{cos\theta_i} - \boldsymbol{cos\theta'_i}| < \boldsymbol{DIFF'} \qquad (19)$$

5 EXPERIMENT

5.1 Experimental setup

To validate the analysis, we compare the modified TSM with TSM and LSM in two cases of environments, called CST1 and CST2. In addition, there are 10 kinds of tests in CST1, and 9 kinds of tests in CST2. There are 1000 kinds of the camera shot images in each test. Note that CST1 and CST2 indicate two courses for the aircraft landing, *i.e.*, the camera shot images indicate different areas between CST1 and CST2. All tests are as shown in Table 1. The two columns in the left side of the table show the names of all tests, and the column in the right side of that shows the detail of each test. In the test of the first row, all craters of the camera shot image have some control errors and some navigation errors. In the test from the 2nd to 9th row, the craters have some errors for taking that image. In the test of the bottom, the craters have some errors for solar altitude.

Table 1. Test pattern format

CST1	CST2	details
400	500	control errors and navigation errors
403	503	lightness
404	504	contrast
405	505	bokeh
406	506	brightness fluctuations
407	507	salt-and-pepper noise
409	509	lense distortion
411	511	image blur
412	512	limb darkening
430		solar altitude

There are five criteria in this experiment, the number of success, miss match, not found, the maximum number of errors of estimation, and the average of the errors.

Table 2. Result of the modified TSM

env	400	403	404	405	406	407	409	411	412	430
ok	999	977	652	579	995	961	998	998	999	761
mm	0	0	2	0	2	8	0	1	0	1
nf	1	23	346	421	3	31	2	1	1	238
avr_d	0.819	0.842	0.887	0.847	0.843	0.953	0.759	0.82	0.802	0.895
max_d	2.807	2.793	3.286	2.807	3.91	5.052	2.858	3.037	2.754	3.656

env	500	503	504	505	506	507	509	511	512
ok	1000	934	449	307	998	701	998	1000	999
mm	0	0	0	0	1	5	0	0	0
nf	0	66	551	693	1	294	2	0	1
avr_d	1.028	1.074	1.15	1.02	1.128	1.16	0.998	1.017	1.025
max_d	2.569	3.288	2.633	2.268	3.283	4.684	2.659	2.385	2.698

Table 3. Result of TSM

env	400	403	404	405	406	407	409	411	412	430
ok	998	981	703	606	998	981	999	999	999	794
mm	2	2	2	1	2	5	1	1	1	7
nf	0	17	295	393	0	14	0	0	0	199
avr_d	1.11	1.12	1.13	1.12	1.12	1.11	1.1	1.11	1.07	1.4
max_d	3.52	3.64	5.43	3.39	3.67	3.89	3.25	3.53	3.39	94.46

env	500	503	504	505	506	507	509	511	512
ok	992	918	459	313	989	737	994	992	995
mm	5	5	2	1	6	11	4	4	2
nf	3	77	539	686	5	252	2	4	3
avr_d	1.47	1.39	1.46	1.43	1.59	1.54	1.48	1.44	1.39
max_d	3.72	3.33	3.28	3.12	3.77	5.06	4.56	3.45	3.12

Table 4. Result of LSM

env	400	403	404	405	406	407	409	411	412	430
ok	1000	984	669	592	998	973	999	999	999	776
mm	0	0	3	0	2	8	1	1	1	4
nf	0	16	328	408	0	19	0	0	0	220
avr_d	0.908	0.912	0.967	0.909	0.907	1.004	0.823	0.905	0.897	0.969
max_d	3.32	3.359	6.761	3.024	4.614	5.147	3.029	3.677	3.817	4.34

env	500	503	504	505	506	507	509	511	512
ok	992	960	494	324	995	763	996	993	993
mm	1	1	4	0	1	13	0	1	0
nf	7	39	502	676	4	224	4	6	7
avr_d	1.165	1.174	1.236	1.127	1.234	1.281	1.109	1.147	1.135
max_d	3.222	5.258	5.898	2.964	3.576	5.993	2.836	3.52	2.874

5.2 Result

Tables 2, 3, and 4 show the result of the three methods, the modified TSM, TSM, and LSM. Each column shows the test patterns, the row indicates the terms of the results: there are the number of success (ok), miss match (mm), not found (nf), the average of the errors (avr_d), and the maximum number of errors of estimation (max_d) in order from the top. In Tab. 1, there are four kinds of cells colored blue, green, yellow, and red. The blue cell indicates that the result is the best among the results in Tabs. 1, 2, and 3. The green cell indicates that the result is better than the result of Eq. 2 or 3. The yellow cell indicates that the result is better than one of other results and worse than the other. The red cell indicates that the result is worst among all results.

The avr_d and the max_d are smallest among almost all test patterns. The results of the avr_d in the modified TSM are worse than those of TSM, but better than those of LSM. Concretely, avr_d of the modified TSM is decreased of 0.073 and 0.112 than TSM in CST1 and CST2, respectively. max_d of that is decreased of 0.813 and 1.08 than TSM in CST1 and CST2, respectively. In addition, avr_d of the modified TSM is decreased of 0.292 and 0.399 than LSM in CST1 and CST2, respectively. max_d of that is decreased of 9.52 and 0.771 than LSM in CST1 and CST2, respectively. The number of the successes is averagely bad, while the number of the miss match is averagely better.

5.3 Discussion

From these results, the modified TSM can estimate the location of the aircraft accurately, but the number of the successes becomes low in CST1. On the other hand, the success rate of the modified TSM becomes high in some tests of CST2. The theoretical analysis is effective for the accurate aircraft localization.

Figure 6 shows the example which the modified TSM can estimate the own location accurately. In this figure, there are several blue and red circles as craters in the crater map and the camera shot image, respectively. Cyan and magenta triangles are matched in the crater map and the camera shot image, respectively. Since there are few craters being in similar position, this situation is difficult to match the craters. The modified TSM can match the similar triangles as those in Figure 6, while TSM cannot match the triangles because it aims to match congruent triangles. This suggests that the modified TSM can estimate the location in the difficult situation as shown in Figure 6.

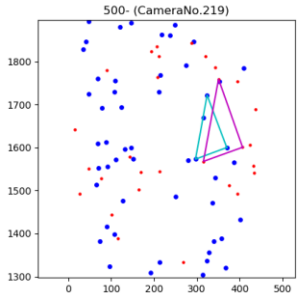

Figure 6. matched example in the modified TSM

However, the modified TSM cannot match the triangles and the craters roughly. Figure 7 shows an example which TSM matched the triangles and the craters. This figure is the same type figure as Figure 6. In this figure, the triangles are very similar, but they are not in the same position. TSM can match the triangles in this situation. Although the result of this situation is miss match in TSM, the difference between true location and the estimated location is under 3.89 (*i.e.*, TSM can estimate accurate enough to land on the planet.) This suggests that the modified TSM can estimate that accurately, but too severe to increase the success rate.

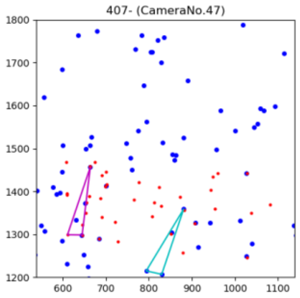

Figure 7. matched example in TSM

4 CONCLUSION

This paper aims to derive advantage of triangle matching method for spacecraft localization by theoretical analysis based on Triangle Similarity Matching (TSM) method, and validates that by the experiment. This paper analyzes the inner and the cross products in the mechanism of TSM. From the theoretical analysis, we can derive two things: (1) if the only craters outside of the triangles are in the different locations, the inner and the cross products' values do not relate on the shape; (2) since the inner and the cross products are influenced of the vector's length, the products should be divided by the length; and (3) the triangles have not to be congruence, but they have to be similar for the advantage of the triangle shape. Furthermore, we improve TSM based on the above findings, and apply it to aircraft localization problems as experiments to validate the effectiveness. In the experiment, we employ two environments and 19 test patterns to show a robustness against any noise. From the experimental results, we revealed those things: (a) the modified TSM can perform accurately than any other methods; (b) the modified TSM can decrease the distance from the true location, averagely 0.3, and max 9.52; and (c) the modified TSM can estimate the location in the difficult situations by matching similar triangles.

This paper shows the advantage of TSM based on theoretical analysis, and the modified TSM based on the analysis can estimate the own location accurately than LSM. However, the number of the successes becomes small in some test patterns, and we cannot prove the advantage yet. In the future, we are going to cope those issues. Concretely, we are going to show the accurate estimation for a few craters for TSM to decrease the number of the required craters. In addition, we are going to focus on the own location calculation process, and analyze this mechanism.

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